

ON THE PREDICTION OF BED EXPANSION FOR LIQUID FLUIDIZED BEDS

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There are several methods available in the fluidization literature for the prediction of the porosity of a liquid fluidized bed of monosized spheres. The majority of practical applications, however, involve non-spherical particles. Furthermore, adding to the complexity of the problem, systems of practical interest usually consist of particles with varying sizes. Granular beds used in water and wastewater filtration is an important example. My goal in this handout is to explain how the expanded height of a liquid fluidized granular bed can be estimated. To this end, the general problem is tackled in three steps, looking at three types of problem starting with the simplest and finishing with the most complex: (1) Beds of monosized spheres. (2) Beds of monosized non-spherical particles (3) Non-uniform beds of non-spherical particles.

MONOSIZED SPHERES

As noted, there are numerous correlations that can be used for this purpose (Couderc, 1985; Hartman *et al.*, 1989; Di Felice, 1995; Epstein, 2003). Akgiray and Soyer (2006) reviewed and compared existing popular correlations and found that an equation they developed yields very accurate predictions:

$$\phi = 3.137 Re_1 + 0.673 Re_1^{1.766} \quad (1)$$

The dimensionless group ϕ and the modified Reynolds number Re_1 are defined as follows (these definitions are valid for both spherical and non-spherical particles; simply set $\psi=1$ in these expressions to restrict attention to spheres):

$$\phi = \frac{\varepsilon^3}{(1-\varepsilon)^2} \frac{\psi^3 d_{eq}^3 \rho (\rho_p - \rho) g}{216 \mu^2} \quad (2)$$

$$Re_1 = \frac{\psi d_{eq} \rho V}{6 \mu (1-\varepsilon)} \quad (3)$$

where

V = velocity based on the empty cross-section of the bed

ε = porosity (i.e. volume fraction of voids) (ε_0 = fixed-bed porosity)

d_{eq} = equivalent diameter defined as the diameter of the sphere with the same volume as a bed particle

ψ = sphericity defined as the surface area of the equivalent volume sphere divided by the actual surface area of the particle

μ = viscosity of the liquid

ρ = density of the liquid

ρ_p = density of the particles

g = gravitational acceleration

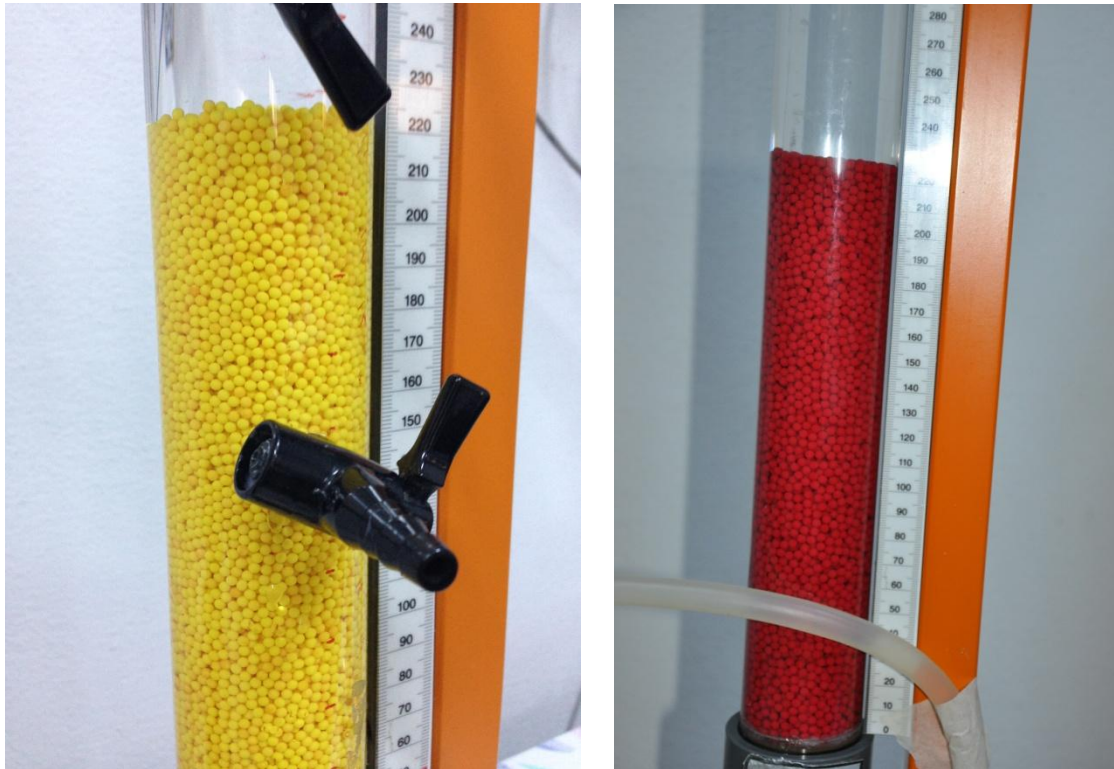


FIGURE 1: Examples of plastic balls (spheres) employed in fluidization experiments carried out at the Marmara University Filtration and Fluidization Research Center (FFRC) laboratories. (Courtesy of Dr. Elif Soyer.)

Two types of problem can be considered: (1) Given the value of velocity, what is the corresponding porosity? (2) Given the desired porosity, what is the corresponding velocity? Usually, instead of the porosity, it is the expanded height L that is of direct practical interest. For either type of problem, the fixed-bed porosity ε_0 and the fixed-bed height L_0 are normally known in advance. Therefore, the bed height L and the porosity ε can be calculated from each other using the following relation:

$$L(1-\varepsilon) = L_0(1-\varepsilon_0) \quad (4)$$

The best way to understand how these equations are used is to look at a specific example.

EXAMPLE 1: Glass balls of 4 mm diameter are fluidized using water at a velocity of 0.20 m/s. What will be the porosity of the bed? Assume density of glass is 2500 kg/m³, density of water is 1000 kg/m³, and viscosity of water is 0.001 N s/m². Assuming $L_0 = 1$ m and $\varepsilon_0 = 0.4$, what will be the expanded bed height and the percent expansion?

Solution:

Using Equations 2 and 3, we write

$$\varphi = \frac{\varepsilon^3}{(1-\varepsilon)^2} \frac{1^3(0.004)^3(1000)(2500-1000)9.81}{216(0.001)^2} = \frac{4360\varepsilon^3}{(1-\varepsilon)^2}$$

$$Re_1 = \frac{1(0.004)(1000)(0.20)}{6(0.001)(1-\varepsilon)} = \frac{133.33}{(1-\varepsilon)}$$

Inserting these into Equation 1, we obtain

$$\frac{4360\varepsilon^3}{(1-\varepsilon)^2} = 3.137 \frac{133.33}{(1-\varepsilon)} + 0.673 \left(\frac{133.33}{(1-\varepsilon)} \right)^{1.766}$$

Since ε appears on both sides of this nonlinear equation, an iterative solution is required. Such a solution is carried out easily using the Solver Add-In in MS-Excel. The result is $\varepsilon = 0.837$ (see the screen images given below). Therefore

$$L = \frac{(1 - \varepsilon_0)L_0}{(1 - \varepsilon)} = \frac{(1 - 0.4)(1m)}{(1 - 0.837)} = 3.68m$$

Percent expansion = $100 (L - L_0)/L_0 = 268 \%$.

Remark: The success of Solver in converging to the correct solution may depend on the initial estimate of the expanded bed porosity. In this example, Solver failed when $\varepsilon = 0.60$ was used as the initial value, but it found the correct solution starting from $\varepsilon = 0.75$ or $\varepsilon = 0.90$. A solution method that is more robust is presented in Example 3.

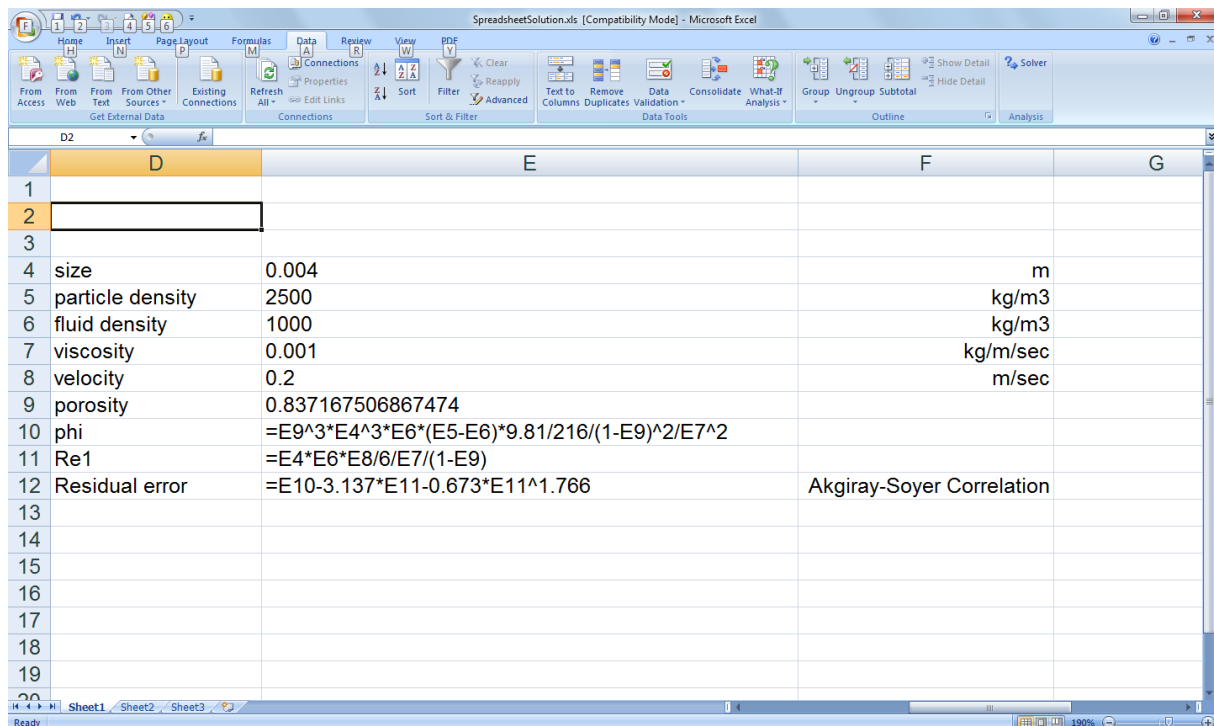


FIGURE 2-a: An Excel worksheet (formulas made visible by clicking the "Show Formulas" button) that can be used for spherical particles (Source: Akgiray and Soyer, 2006).

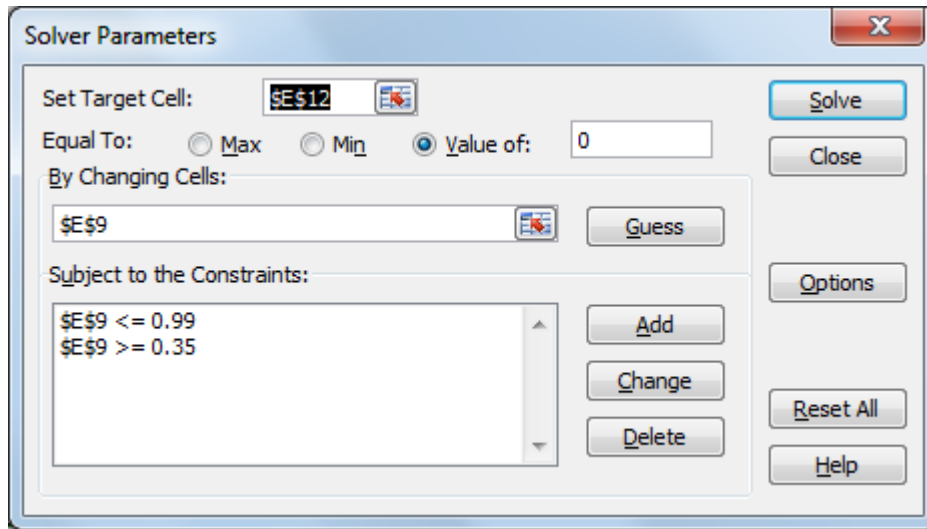


FIGURE 2-b: The use of the Solver Add-In in MS-Excel. The constraints $0.35 \leq \varepsilon$ and $\varepsilon \leq 0.99$ were imposed to assure a physically meaningful solution. The addition of such constraints is normally not required, but may be useful if the value of ε becomes negative or greater than unity during iterations.

MONOSIZED NON-SPHERICAL PARTICLES

There are very few correlations applicable to non-spherical particles and, as a matter of fact, a vast majority of the literature equations are restricted to spheres. Soyer and Akgiray (2009) carried out extensive tests with non-spherical filter media and extended Equation 1 to handle both spherical and non-spherical particles. The generalized equation is as follows:

$$\log \varphi = \log(3.137 \text{Re}_1 + 0.673 \text{Re}_1^{1.766}) - (0.930 + 0.274 \log \text{Re}_1)(-\log \psi)^{1.262} \quad (5)$$

It may be noted that Equation 5 becomes identical with Equation 1 when $\psi = 1$.

EXAMPLE 2: A sieved fraction of silica sand used in fluidization tests had the following properties: $d_{eq} = 1.40$ mm, $\rho_p = 2627.6$ kg/m³, $\psi = 0.714$. Initial bed depth was $L_0 = 22.9$ cm and initial porosity was $\varepsilon_0 = 0.466$. The bed was fluidized using water at 25.2 °C. Estimate the porosity, expanded height and percent expansion of the bed as a function of velocity in the range 0.02 m/s to 0.07 m/s. At this temperature, $\rho = 997.0$ kg/m³, $\mu = 0.000886$ N s/m².

Solution:

First, using Equations 2 and 3, we write

$$\varphi = \frac{\varepsilon^3}{(1 - \varepsilon)^2} \frac{(0.714)^3 (0.0014)^3 (997)(2627.6 - 997)9.81}{216(0.000886)^2} = \frac{93.94\varepsilon^3}{(1 - \varepsilon)^2}$$

$$\text{Re}_1 = \frac{(0.714)(0.0014)(997)V}{6(0.000886)(1 - \varepsilon)} = \frac{187.47V}{(1 - \varepsilon)}$$

Also

$$(-\log \psi)^{1.262} = (-\log 0.714)^{1.262} = 0.0884$$

Inserting these into Equation 5, we obtain

$$\log\left(\frac{93.94\varepsilon^3}{(1-\varepsilon)^2}\right) = \log\left(3.137\frac{187.47V}{(1-\varepsilon)} + 0.673\left(\frac{187.47V}{(1-\varepsilon)}\right)^{1.766}\right) + \left[0.93 + 0.274\log\left(\frac{187.47V}{(1-\varepsilon)}\right)\right](0.0884)$$

This equation is to be solved to find ε for each given value of V . While such calculations could be carried out manually, it is much faster and easier to use a computer for this purpose. To solve this problem and other problems similar to it, the worksheet of Figure 2 was generalized to handle non-spherical media. The resulting worksheet is shown in Figure 3-a. Solver is executed once for each different value of V . The results are shown in Table 1 and illustrated in Figure 4. Also shown in Figure 4 are the experimental data for this system (Source: Soyer and Akgiray, 2009).

	A	B	C
1			
2			
3	sphericity	0.714	
4	size (deq)	0.0014	m
5	particle density	2627.6	kg/m3
6	fluid density	997	kg/m3
7	viscosity	0.000886	kg/m/sec
8	velocity	0.02	m/sec
9	porosity	0.468512734089273	
10	phi	=B9^3*(B3*B4)^3*B6*(B5-B6)*9.81/216/(1-B9)^2/B7^2	
11	Re1	=B3*B4*B6*B8/6/B7/(1-B9)	
12	Residual error	=LOG10(B10)-LOG10(3.137*B11+0.673*B11^1.766)+(0.93+0.274*LOG10(B11))*(-LOG10(B3))^1.262	
13	L0	0.229	m
14	ε0	0.466	
15	L	=B13*(1-B14)/(1-B9)	m
16	Percent expansion	=100*(B15-B13)/B13	
17			
18			
19			
20			
21			
22			
23			
24			

FIGURE 3-a: An Excel worksheet (formulas made visible by clicking the “Show Formulas” button) that can be used for both spherical and non-spherical particles. This image was captured after the solution ($\varepsilon = 0.469$) was found. A reasonable initial value for ε , e.g. a value between ε_0 and 0.90 should be entered before Solver is executed.

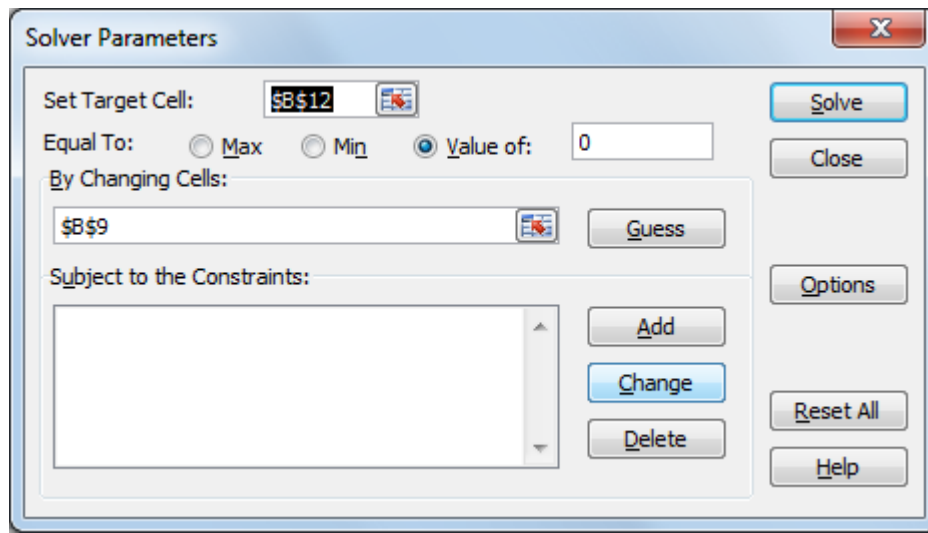


FIGURE 3-b: The use of the Solver Add-In in MS-Excel.

Table-1: Calculated results for Example 2.

velocity (m/sec)	porosity	L (m)	Percent Expansion
0.02	0.469	0.230	0.6
0.03	0.547	0.270	17.9
0.04	0.613	0.316	38.0
0.05	0.669	0.369	61.3
0.06	0.718	0.434	89.4
0.07	0.761	0.512	123.4

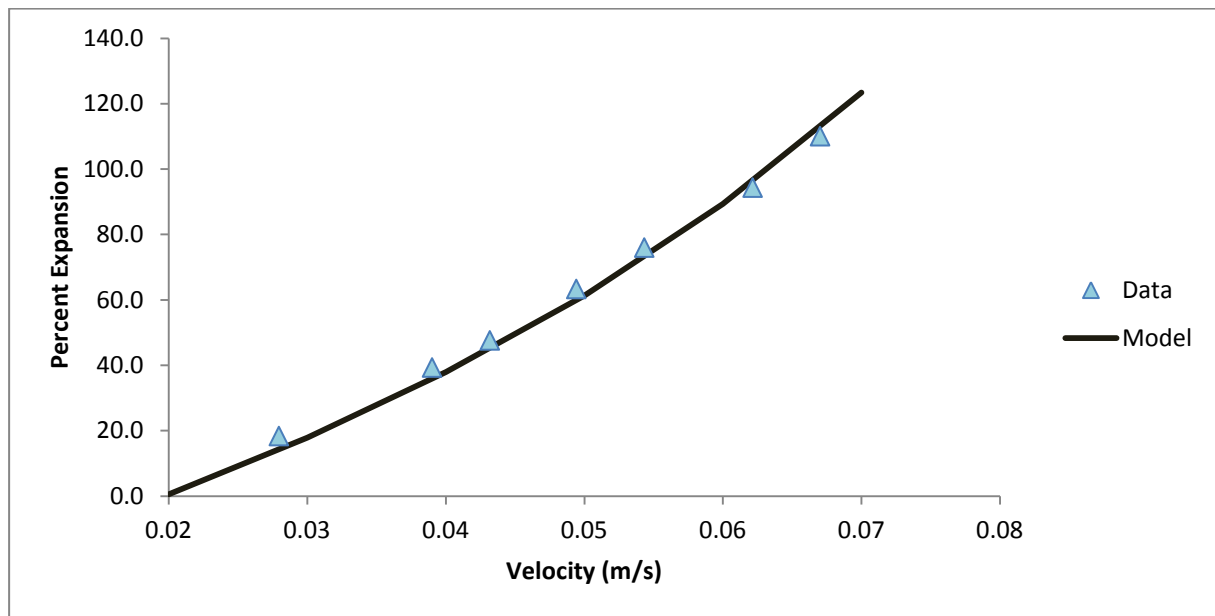


FIGURE 4: Model predictions versus experimental data for Example 2.

NON-UNIFORM BEDS OF NON-SPHERICAL PARTICLES

The majority of real filter media, in addition to being non-spherical in shape, contain a distribution of sizes. In practice, a sieve analysis is used to characterize the size distribution of a non-uniform granular bed. The “serial model” is currently recommended to predict the total expansion of such a bed: A bed with a size gradation is considered to consist of several layers of approximately uniform size according to the sieve analysis data, and the expansion of each layer is separately calculated. The total expansion is calculated by adding the expansions of all the layers (Fair et al., 1971; AWWA, 1999). This calculation method may also be used to predict the total expansion of a multimedia bed.

EXAMPLE 3: (Adopted from Peavy *et al.*, Example 4-14) Water at 20 °C is used to backwash a filter bed at a rate of 0.015 m/s. The fixed bed is 0.75 m deep and contains nonuniform sand (specific gravity of 2.65). The fixed-bed porosity and sphericity are 0.4 and 0.85, respectively, throughout the depth of the bed. Determine the depth of the expanded bed. The size distribution of the grains is given below.

Table-2: Sieve analysis for the filter bed of Example 3.

Sieve Size (USA mesh)	Weight fraction	Retaining size (mm)	Passing size (mm)
14 (retaining)	0.01	1.41	
14-20	0.11	0.84	1.41
20-25	0.2	0.71	0.84
25-30	0.32	0.6	0.71
30-35	0.21	0.5	0.6
35-40	0.13	0.42	0.5
40 (passing)	0.02		0.42

Solution:

This problem can be solved using the worksheet of Example 2 (see Figure 3). Instead of using different velocities as in Example 2, however, velocity is kept constant at 0.015 m/s, but particle size is varied. A different size (d_{eq}) is used for each fraction. This bed is assumed to consist of seven uniform layers and the fixed-bed depth of each layer is assumed to be proportional to its weight fraction. For the 25x30 mesh fraction, for example, fixed-bed depth is calculated as $0.32 \times 0.75 \text{ m} = 0.24 \text{ m}$ (see cell I17 in Figure 7). The fixed-bed depths of other fractions are similarly calculated and displayed in Column I (see Figures 7 and 8).

The topmost sieve retains particles that cannot pass through 1.41 mm openings. Since there is no sieve above this sieve, no passing size is specified. The weight fraction (i.e. 0.01) of grains retained on this sieve, however, is very small and little error is introduced by assuming a passing size equal to the retaining size for this fraction. Similarly, grains passing through the 0.42 mm sieve have no retaining size (they accumulate on the tray at the bottom of the sieve assembly). Again, using a retaining size equal to the passing size should introduce a negligible error in the overall calculation, as the weight fraction (0.02) of the grains falling all the way down to the bottom pan (tray) is quite small.



FIGURE 5: Sieves placed in the shaking device at the Marmara University FFRC laboratories. There are six sieves and a bottom pan in this picture. The example problem also involves six sieves plus a bottom pan. (Courtesy of Ms. Selda Yiğit Huncce.)



FIGURE 6: One of the sieves of Figure 5. A crushed glass sample was being sieved manually when this picture was taken (Courtesy of Dr. Elif Soyer.)

Equivalent diameters are not reported in this example. (See Soyer and Akgiray (2009) for a description of how d_{eq} is measured.) A common –but not very good- approximation is to use the arithmetic average of the passing and retaining sizes. This approach is adopted in the worksheet of Figures 7 and 8 (see the formulas used in Column F).

At this point, the information in Columns B, C, D, E, F, G and I have been typed in or calculated and the next step is the calculation of expanded porosity for each sieve fraction. Solver can be executed seven times to find the expanded porosities of the seven fractions (the worksheet in Figure 3 can be used for this purpose). The expanded depths of all fractions are then calculated and summed to find the total expanded depth of the bed. The calculations in Column J are automatic (see the formulas shown in Figure 8) once the expanded porosities are calculated.

A better alternative to the use of Solver is to write a function in VBA to carry out the iterations. This can speed up calculations because one would not have to repeat the steps needed to execute Solver for each different fraction (or, for each different velocity as in Example 2). Copying and pasting the relevant function call (which is written as an Excel formula) from one cell to others would be sufficient. Another advantage of this approach is that, one can change any of the input parameters (e.g. temperature, particle density, weight fractions etc.) and then just press the “Enter” key and the results will be automatically calculated. With the first approach, one would have to execute Solver seven additional times for each change in any one of the input parameters. The Appendix gives the full listing of a VBA code that I wrote just for this purpose. The program in the Appendix seems to work correctly and hopefully is easy enough to understand (and to adapt and further develop) by others who have some programming skills. Just one warning: This program has not been tested much (it was developed within a few hours and tested only with the examples in this handout); so if you find what you think is an error or a weakness please let me know.

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2													
3		Example 4-14 in Peavy et al., p. 173											
4													
5		L0		0.75 m									
6		velocity		0.015 m/sec									
7		temperature		20									
8		viscosity		1.00E-03 kg/m/sec									
9		fluid density		998.2 kg/m3									
10		particle density		2650 kg/m3									
11		fixed-bed porosity		0.4									
12													
13		Sieve Size (USA mesh)	Weight fraction	Retaining size (mm)	Passing size (mm)	deq (mm)	ψ	ϵ	L0	L			
14		14	0.01	1.41	1.41	1.41	0.85	0.409	0.0075	0.008			
15		14-20	0.11	0.84	1.41	1.13	0.85	0.456	0.0825	0.091			
16		20-25	0.2	0.71	0.84	0.78	0.85	0.542	0.15	0.196			
17		25-30	0.32	0.6	0.71	0.66	0.85	0.583	0.24	0.346			
18		30-35	0.21	0.5	0.6	0.55	0.85	0.628	0.1575	0.254			
19		35-40	0.13	0.42	0.5	0.46	0.85	0.674	0.0975	0.180			
20		40	0.02	0.42	0.42	0.42	0.85	0.698	0.015	0.030			
21													
22													
23													
24													
									Ltotal		1.10 m		
									Percent expansion		47.2		

FIGURE 7: Results for Example 3.

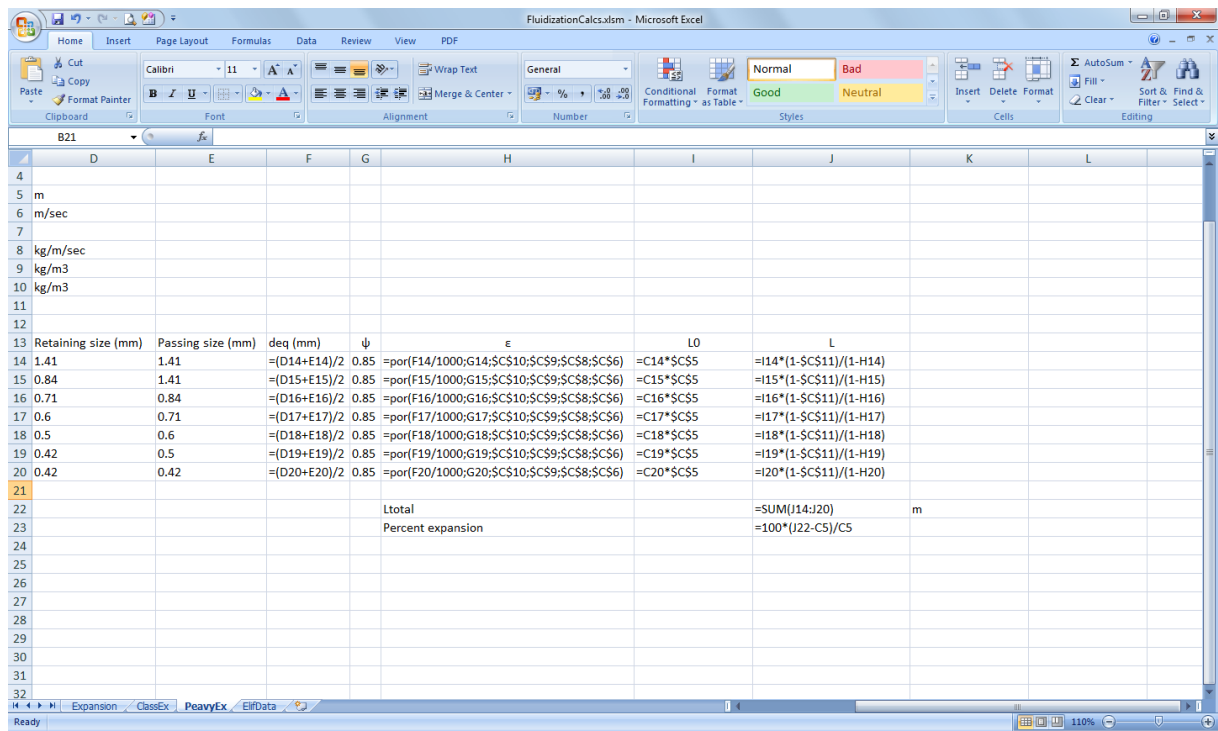


FIGURE 8: The Excel formulas used in the worksheet shown in Figure 7.

APPENDIX

```
'Program to calculate fluidized bed expansion
'Author: Ömer Akgiray, Marmara University
'Date: May 20, 2014
```

```
Option Explicit
Dim Ga As Double    'Galileo number
Dim Re As Double    'Reynolds number
Dim Sph As Double    'Sphericity (same as psi; different name is used here to avoid conflict with psi passed as argument)
```

Function log10(x As Double) As Double

```
    log10 = Application.log10(x)
End Function
```

Function logPhi(Re1 As Double, psi As Double) As Double

```
    'Correlation by Soyer and Akgiray(2009)
    'Re1 = modified Reynolds number, psi = sphericity
    logPhi = log10(3.137 * Re1 + 0.673 * Re1 ^ 1.766)
        - (0.93 + 0.274 * log10(Re1)) * (-log10(psi)) ^ 1.262
End Function
```

Function Residual(e As Double) As Double

```
    'Function calculates residual error
    'Root finding routines (e.g. Bisection) try to find e such that Residual = 0
    'e = porosity of fluidized bed
    Dim Re1 As Double, phi As Double
    Re1 = Re / 6 / (1 - e)
    phi = e ^ 3 / (1 - e) ^ 2 * Sph ^ 3 * Ga / 216
    Residual = log10(phi) - logPhi(Re1, Sph)
End Function
```

Function por(deq As Double, psi As Double, densp As Double, _ densf As Double, visc As Double, vel As Double) As Double

```
    Dim e As Double, Re1 As Double, phi As Double, nb As Integer
    Dim e1() As Double, e2() As Double, Found As Boolean, Error As Double
    Const TOL = 0.00000001 'Tolerance for error
    Re = psi * deq * densf * vel / visc
    Ga = deq ^ 3 * densf * (densp - densf) * 9.81 / visc ^ 2
    Sph = psi
    'We will allow porosities between 0.00001 and 0.99999, but
    'if the porosity is less than fixed-bed porosity, the bed is
    'not fluidized and the answer found here should be replaced by
    'fixed-bed porosity; if porosity is higher than 0.90, then we are
```

```

'outside the range of applicability of Phi versus Re1 correlations
'and the answer may be inaccurate and should be used with caution
'Now, bracket one root (we expect only one root)
nb = 1
zbrak 0.00001, 0.99999, 100, nb, e1, e2, "Residual"
If nb = 0 Then 'nb = number of roots found between 0.00001 and 0.99999
    'nb = 0 means there is no solution for the given input parameters
    'This may happen if velocity is too high (greater than terminal settling velocity)
    por = 0
    Exit Function
End If
'Find the root between e1(1) and e2(1)
Bisect e1(1), e2(1), e, Error, TOL, Found, "Residual"
If Found Then
    por = e
Else
    por = 0
End If
End Function

```

```

Sub Bisect(ByVal x1 As Double, ByVal x2 As Double, _
    ByRef XMid As Double, ByRef FMid As Double, _
    ByVal TOL As Double, ByRef Found As Boolean, _
    ByVal Func As String)
    Dim Itr As Integer
    Const MAXItr = 50
    Dim A As Double, B As Double, FA As Double, FB As Double
    Found = True
    A = x1
    B = x2
    FA = Application.Run(Func, A)
    FB = Application.Run(Func, B)
    If FA * FB > 0 Then
        Found = False
        Exit Sub
    End If
    For Itr = 1 To MAXItr
        XMid = (A + B) / 2
        FMid = Application.Run(Func, XMid)
        If FA * FMid <= 0# Then
            B = XMid
            FB = FMid
        Else
            A = XMid

```

```

        FA = FMid
    End If
    If Abs(B - A) < TOL Then
        Exit Sub
    End If
Next Itr
Found = False
End Sub

```

'The following routine finds brackets containing zero crossings
 'Each such bracket can be used as input to Subroutine Bisect
 'See Numerical Recipes in Fortran 77, 2nd Ed., pp.343-345.

```

Sub zbrak(ByVal x1 As Double, ByVal x2 As Double, _
    ByVal n As Integer, ByRef nb As Integer, _
    ByRef xb1() As Double, ByRef xb2() As Double, _
    ByVal Func As String)
    'x1, x2: interval end points
    'n : the interval [x1,x2] is divided into n equal segments
    'nb: maximum number of roots sought within [x1,x2]
    '    on return, nb is the number of roots (zero crossings) found
    'xb1, xb2: bracketing pairs found
    Dim nbb As Integer, x As Double, dx As Double, fp As Double, i As Integer, fc As Double
    nbb = 0
    x = x1
    dx = (x2 - x1) / n
    fp = Application.Run(Func, x)
    For i = 1 To n
        x = x + dx
        fc = Application.Run(Func, x)
        If fc * fp <= 0 Then
            nbb = nbb + 1
            ReDim Preserve xb1(1 To nbb), xb2(1 To nbb)
            xb1(nbb) = x - dx
            xb2(nbb) = x
            If nbb = nb Then
                nb = nbb
                Exit Sub
            End If
        End If
        fp = fc
    Next i
    nb = nbb
End Sub

```

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