ENVE 2061 BASIC FLUID MECHANICS

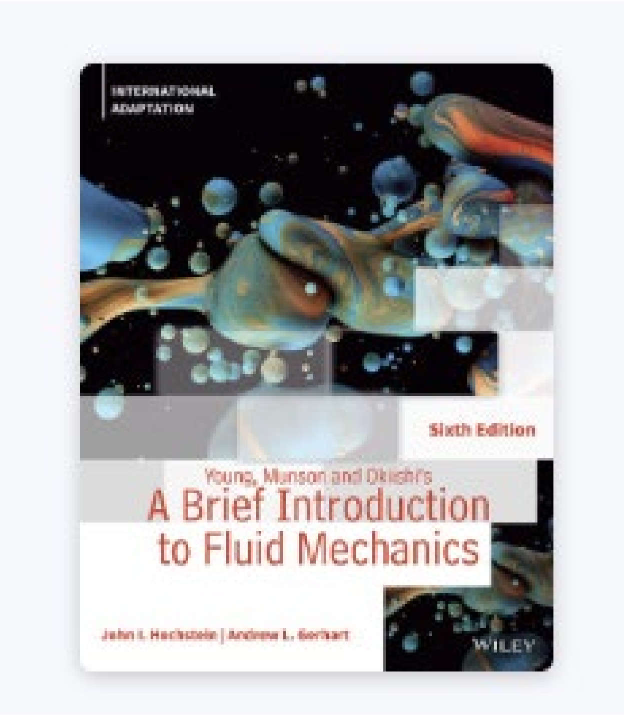
HYDROSTATIC FORCES

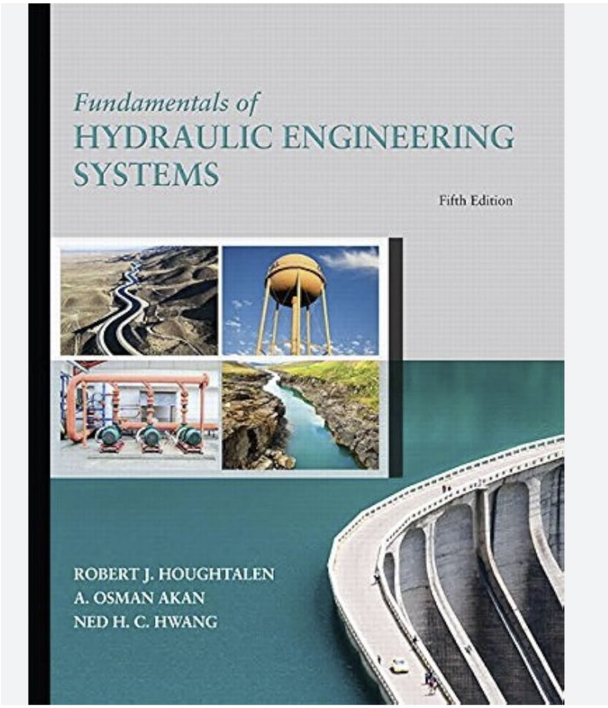
Assoc. Prof. Neslihan Semerci

Lecture Notes from

- A Brief Introduction to Fluid Mechanics, 6th Edition, Donald F. Young, Bruce R. Munson, Theodore H. Okiishi, Wade W. Huebsch

- Fundamentals of Hydraulic Engineering Systems, 5th Edition, Houghtalen R. J., Akan, O. A., Hwang, N. H. C.





Learning Objectives:

- calculate the hydrostatic pressure force on a plane or curved submerged surface.

Forces due to Static Fluids

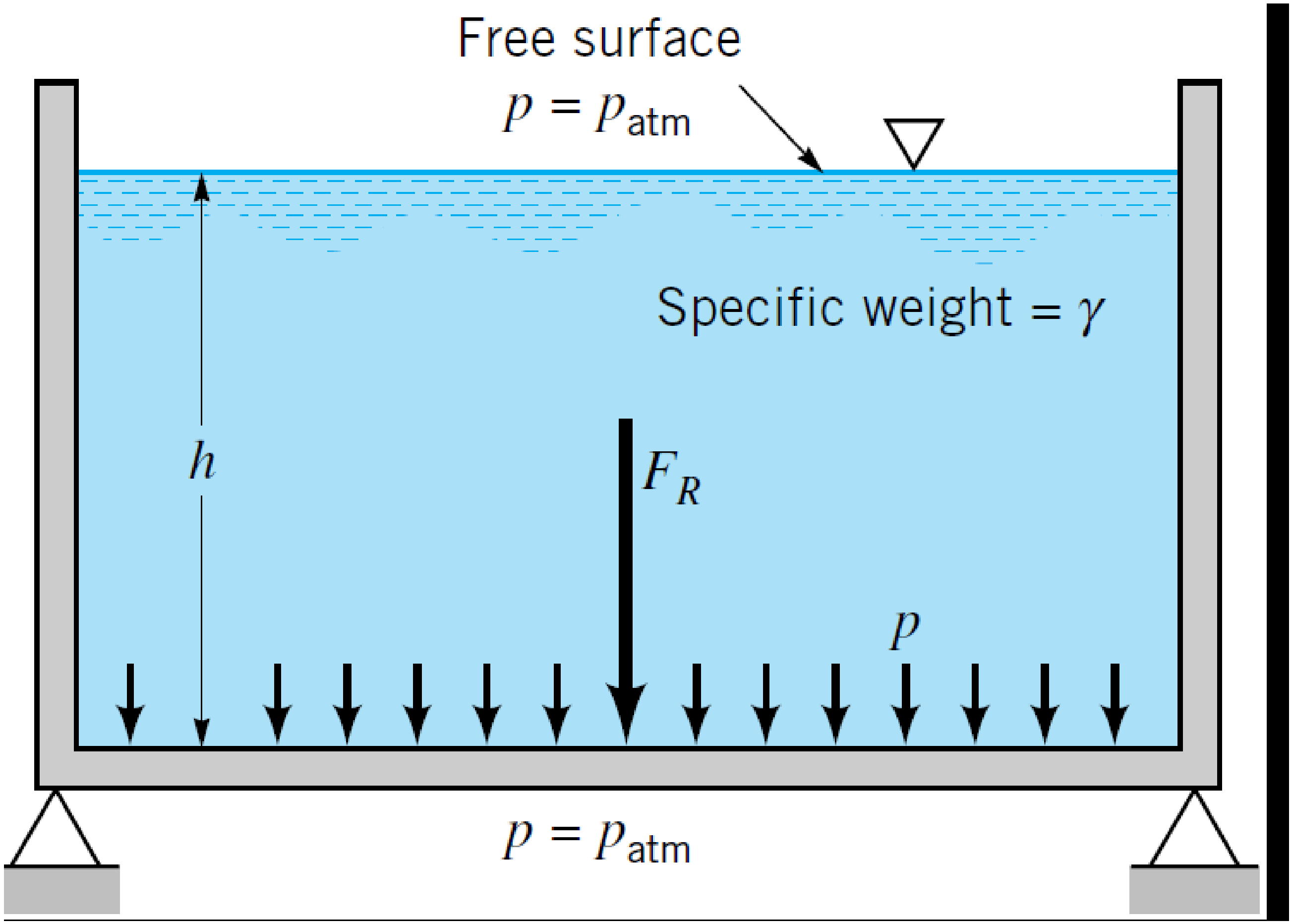
$$F=P.A$$

This equation is applied directly only when the pressure is uniform over the entire area of interest.

1) when the fluid is a gas ➔ the pressure to be equal throughout the gas because of its low specific weight
2) If the area of the surface is small ➔ reasonable to ignore the variation of the pressure over the face of the piston
3) If the surface is flat, horizontal surface as on the bottom of the tanks.

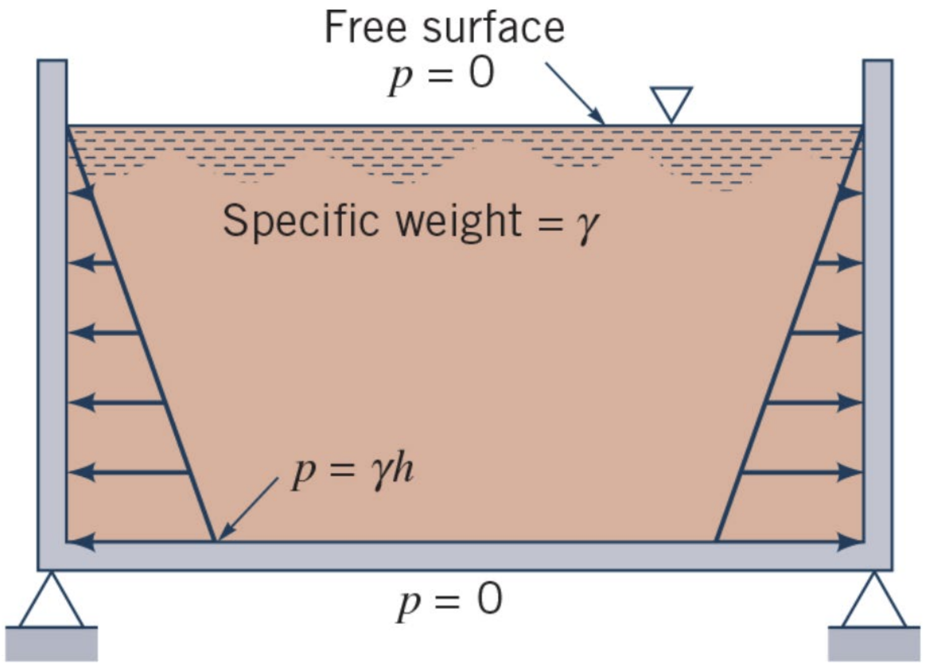
In other cases; when the surface is **vertical**, **inclined**, or **curved**, the variation of pressure with depth and special analysis approaches should be taken into account.

HYDROSTATIC FORCE ON PLANE SURFACES



Pressure is constant & uniformly distributed over the bottom: so the resultant force acts through the centroid of the area

HYDROSTATIC FORCE ON PLANE SURFACES



(b) Pressure on tank ends

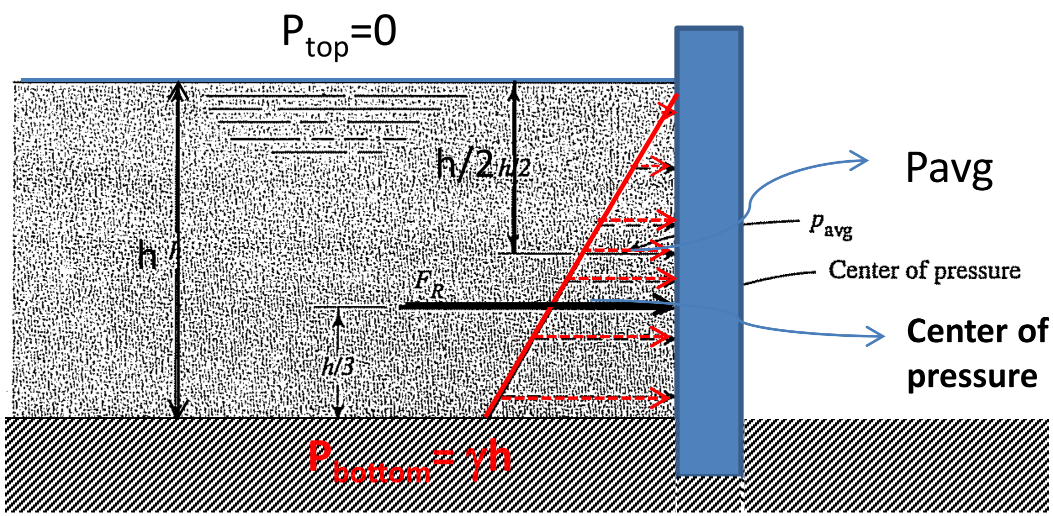
Pressure on the walls of the tank is not uniformly distributed.

$\bar{h}$=Vertical distance from the fluid surface to the centroid of the area
γ= Specific weight of the fluid
A= Area of the surface

$$\mathbf{F} = \gamma \bar{h} A$$

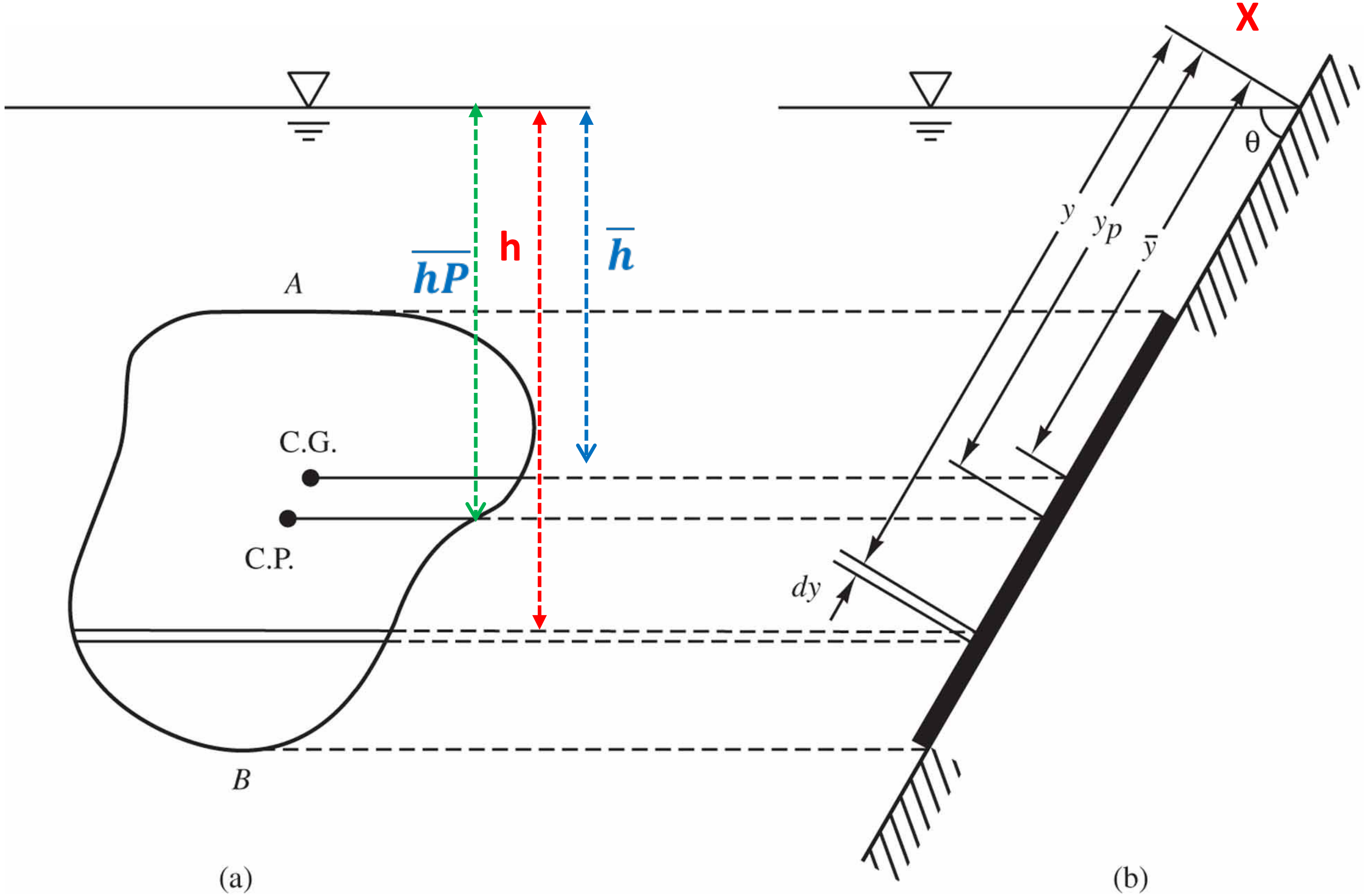
HYDROSTATIC FORCES ON FLAT SURFACES

- The actual force is distributed over the entire surface.



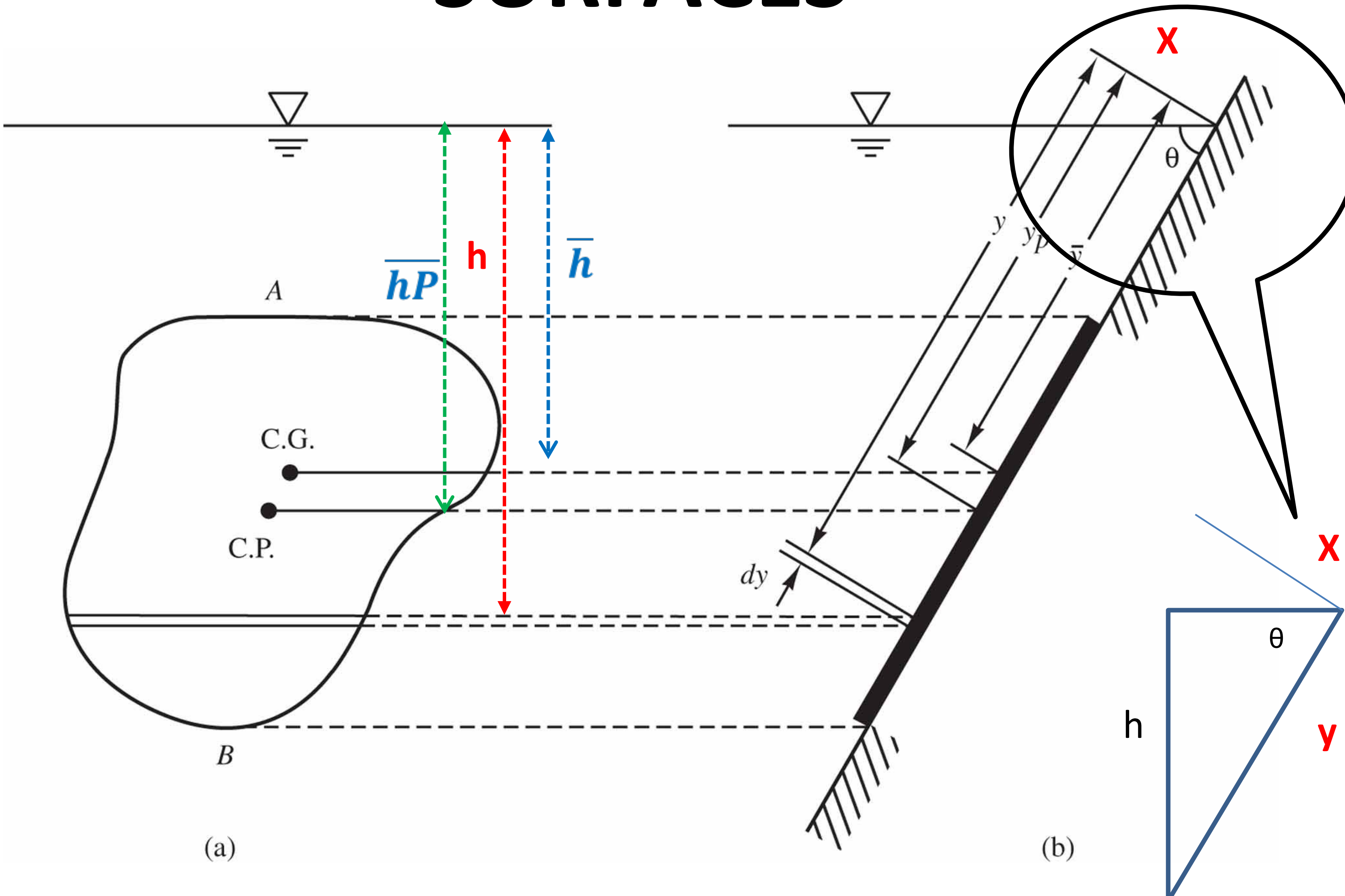
- For the purpose of the analysis, it is desirable to determine the **resultant force** and the place where it acts, called «**center of pressure**».
- If the entire force were concentrated at a single point, where would that point be and what would the magnitude of the force be ?

HYDROSTATIC FORCES ON FLAT SURFACES



$$P = \gamma_{liquid} h = \gamma y \sin\theta$$

HYDROSTATIC FORCES ON FLAT SURFACES



$$P = \gamma_{liquid} h = \gamma y \, \text{Sin}\theta$$

HYDROSTATIC FORCES ON FLAT SURFACES

- The total force on the strip is the pressure times the area

$$dF=\gamma y\ \mathrm{Sin}\theta dA$$

- The total pressure force over the entire AB plane surface is the sum of pressure on all strips.

$$F = \int_A dF = \int_A \gamma y \sin\theta\, dA = \gamma \sin\theta \int_A ydA$$

$$\mathbf{F=}\ \gamma \sin\theta A\bar{y}$$

Centroid, center of gravity & center of pressure

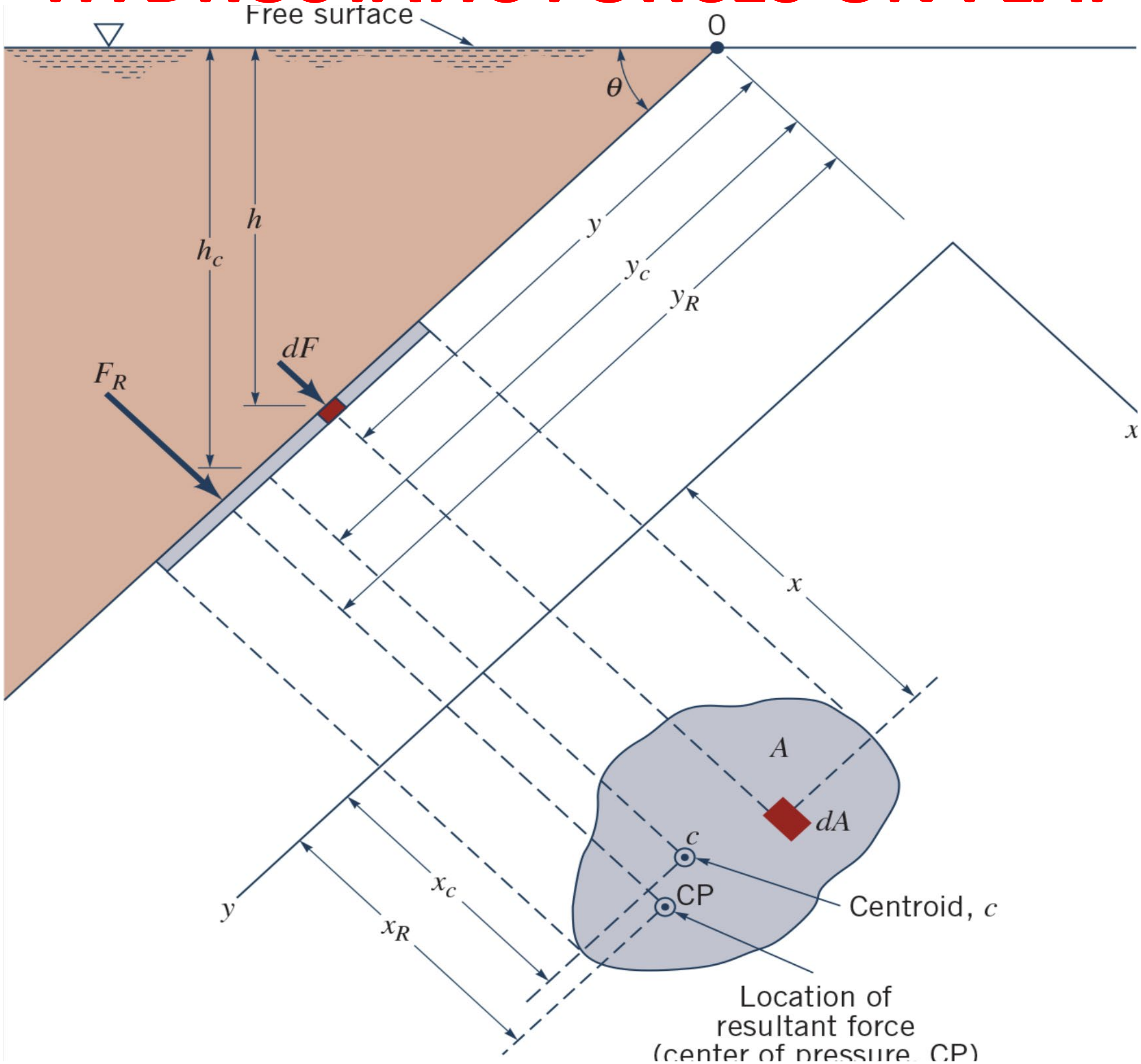
$\bar{y}$ = distance measured from the x-axis to the centroid (or the center of gravity) of the AB plane.

$$F = \gamma \sin\theta A \bar{y} \qquad F = \gamma \bar{h} A$$

Total Hydrostatic Pressure Force on any submerged plane surface is equal to the product of the surface area and the pressure acting at the centroid of the plane surface

$\bar{h}$=vertical distance measured from the water surface to the centroid of the AB plane.

HYDROSTATIC FORCES ON FLAT SURFACES



centroid or center of gravity

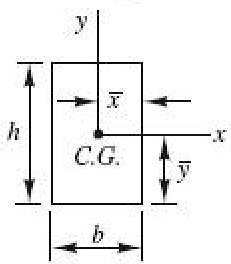
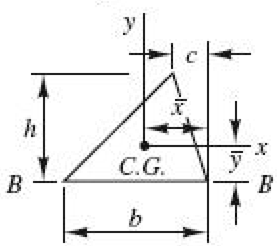
centroid : In physics, the word **centroid** means the geometric center of the object's shape, as above, but **barycenter** may also mean its physical center of mass or the center of gravity, depending on the context.

Informally, the center of mass (and center of gravity in a uniform gravitational field) is the average of all points, weighted by the local density or specific weight. If a physical object has uniform density, then its center of mass is the same as the centroid of its shape.

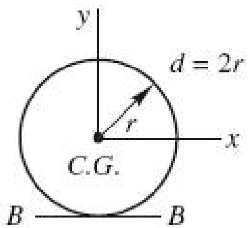
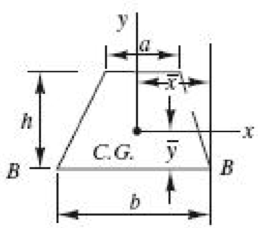
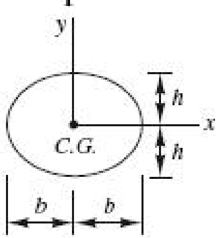
centroid or center of gravity

Table 2.1 Surface Area, Centroid, and Moment of Inertia of Certain Simple Geometrical Plates

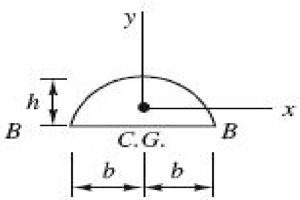
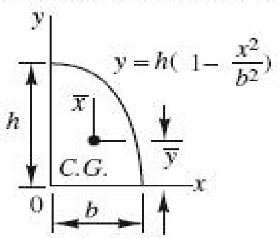
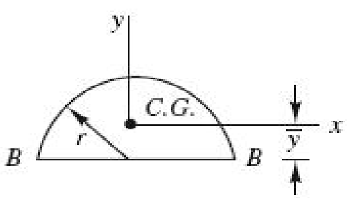
TABLE 2.1 Surface Area, Centroid, and Moment of Inertia of Certain Simple Geometrical Plates

Shape	Area	Centroid	Moment of Inertia About the Neutral x-Axis
Rectangle 	bh	$\bar{x} = \frac{1}{2}b$ $\bar{y} = \frac{1}{2}h$	$I_0 = \frac{1}{12}bh^3$
Triangle 	$\frac{1}{2}bh$	$\bar{x} = \frac{b + c}{3}$ $\bar{y} = \frac{h}{3}$	$I_0 = \frac{1}{36}bh^3$

centroid or center of gravity

Circle 	$\frac{1}{4}\pi d^2$	$\bar{x} = \frac{1}{2}d$ $\bar{y} = \frac{1}{2}d$	$I_0 = \frac{1}{64}\pi d^4$
Trapezoid 	$\frac{h(a+b)}{2}$	$\bar{y} = \frac{h(2a+b)}{3(a+b)}$	$I_0 = \frac{h^3(a^2+4ab+b^2)}{36(a+b)}$
Ellipse 	πbh	$\bar{x} = b$ $\bar{y} = h$	$I_0 = \frac{\pi}{4}bh^3$

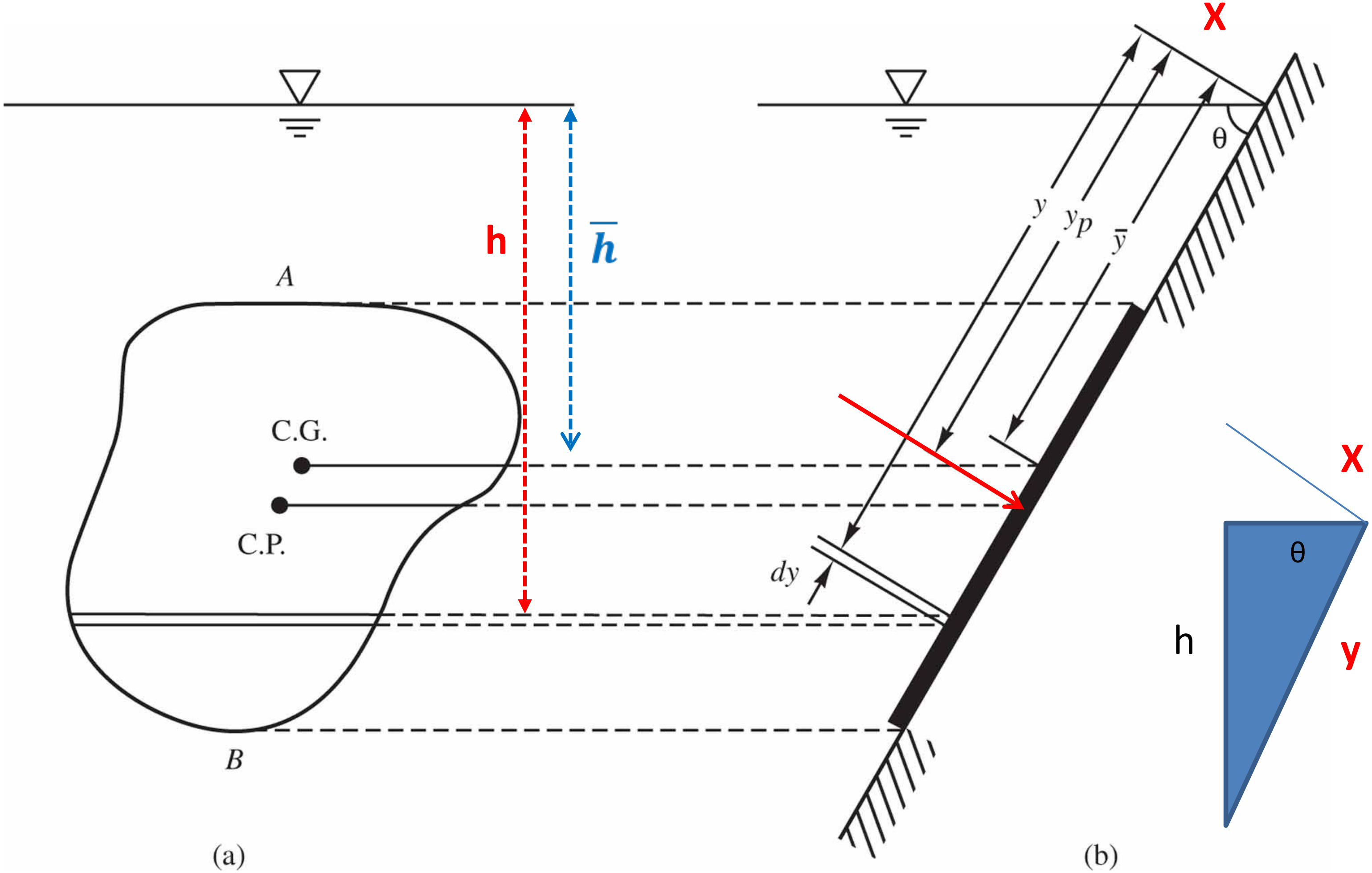
centroid or center of gravity

Shape	Area	Centroid	Moment of Inertia About the Neutral x-Axis
Semi-ellipse 	$\frac{\pi}{2}bh$	$\overline{x} = b$ $\overline{y} = \frac{4h}{3\pi}$	$I_0 = \frac{(9\pi^2 - 64)}{72\pi}bh^3$
Parabolic section 	$\frac{2}{3}bh$	$\overline{y} = \frac{2}{5}h$ $\overline{x} = \frac{3}{8}b$	$I_0 = \frac{8}{175}bh^3$
Semicircle 	$\frac{1}{2}\pi r^2$	$\overline{y} = \frac{4r}{3\pi}$	$I_0 = \frac{(9\pi^2 - 64)r^4}{72\pi}$

Center of pressure

- Pressure forces acting on a plane are distributed over every part of the surface
- They are parallel and acts normal to the surface
- These parallel forces can be analytically repplaced by a single «**single resultant force F**»
- The resultant force acts normal to the surface
- The point on the plane surface at which the resultant force acts is known as the center of pressure.

HYDROSTATIC FORCES ON FLAT SURFACES



$$P = \gamma_{liquid} h = \gamma y \, Sin\theta$$

Center of pressure

- yp: distance measured from the x-axis to the center of pressure

$$Fy_P = \int_A ydF = Fy_P = \int_A y\underbrace{\gamma h dA}_{dF}$$

$$y_P = \frac{\int_A ydF}{F} = \frac{\int_A y(\gamma y \mathrm{Sin}\theta dA)}{\gamma \mathrm{Sin}\theta A\bar{y}}$$

$$y_P = \frac{\int_A y\cancel{\gamma} y\cancel{\mathrm{Sin}\theta} dA}{\cancel{\gamma \mathrm{Sin}\theta} A\bar{y}} = \frac{\int_A y^2 dA}{A\bar{y}}$$

- $\int_A y^2 dA = I_x \rightarrow$ Moment of inertia with respect to x-axis
- $A \times \bar{y} = M_x \rightarrow$ static moment of the plane are with respect to x-axis

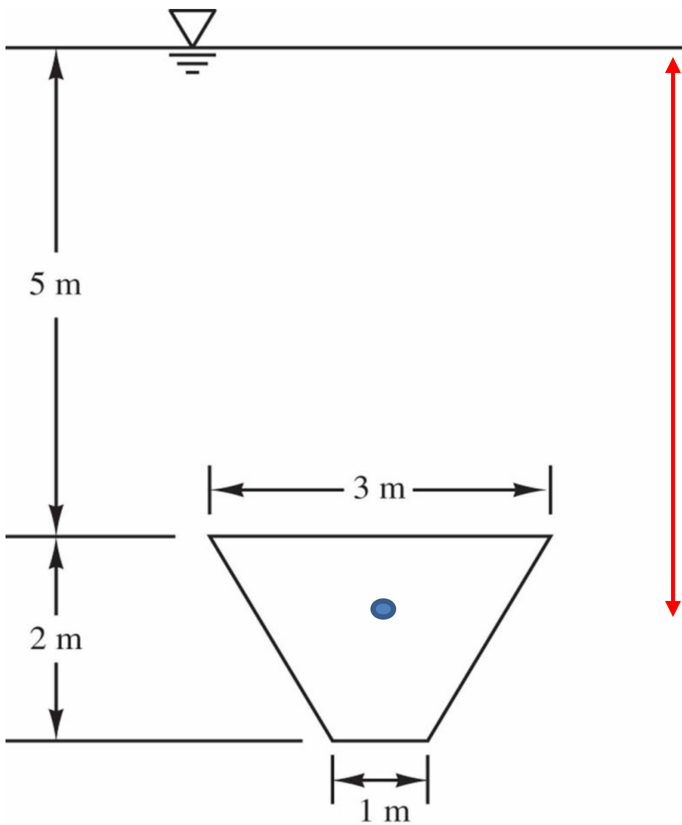
$$y_P = \frac{I_x}{M_x}$$

$y_P = \frac{I_0 + A\bar{y}^2}{A\bar{y}} = \frac{I_0}{A\bar{y}} + \bar{y}$ (with respect to the centroid of the plane)

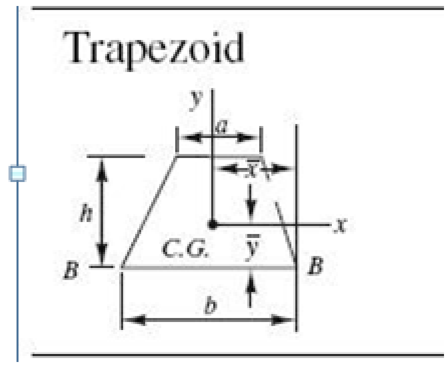
Centroid & center of pressure

The center of pressure of any submerged plane surface is always below the centroid of the surface area.

- **Example 15.1.(Hwang et al., 4th Edition)** A vertical trapezoidal gate with its upper edge located 5 m below the free surface of water is shown in Figure 15.1. Determine the total pressure force and center of pressure on the gate.



$$\bar{h} = 5.83\ m$$



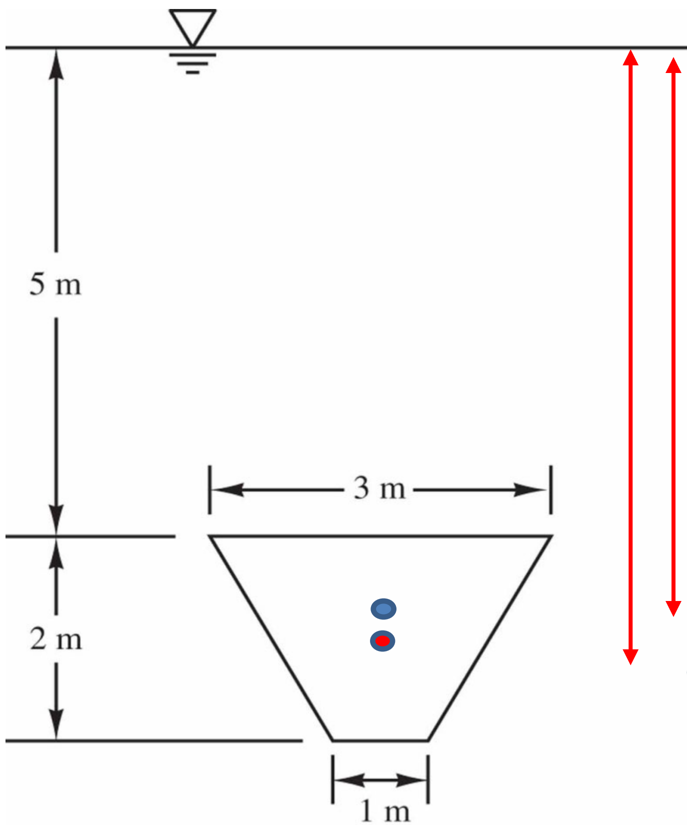
$$\bar{y} = \frac{h(2a + b)}{3(a + b)}$$

a= 1 m
b = 3 m

$$\bar{y} = 0.83\ m$$

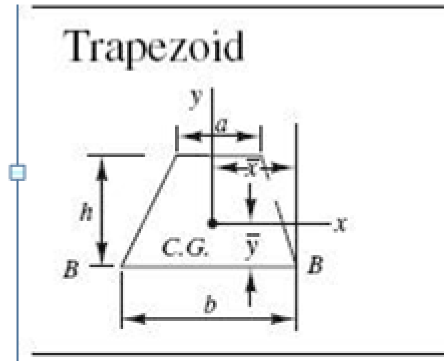
$$\bar{h} = h + \bar{y}$$

- **Example 15.1.(Hwang et al., 4th Edition)** A vertical trapezoidal gate with its upper edge located 5 m below the free surface of water is shown in Figure 15.1. Determine the total pressure force and center of pressure on the gate.



$\overline{h} = 5.83\ m$

$yp = 5.88\ m$



$$I_0 = \frac{h^3(a^2 + 4ab + b^2)}{36(a + b)}$$

a= 1 m

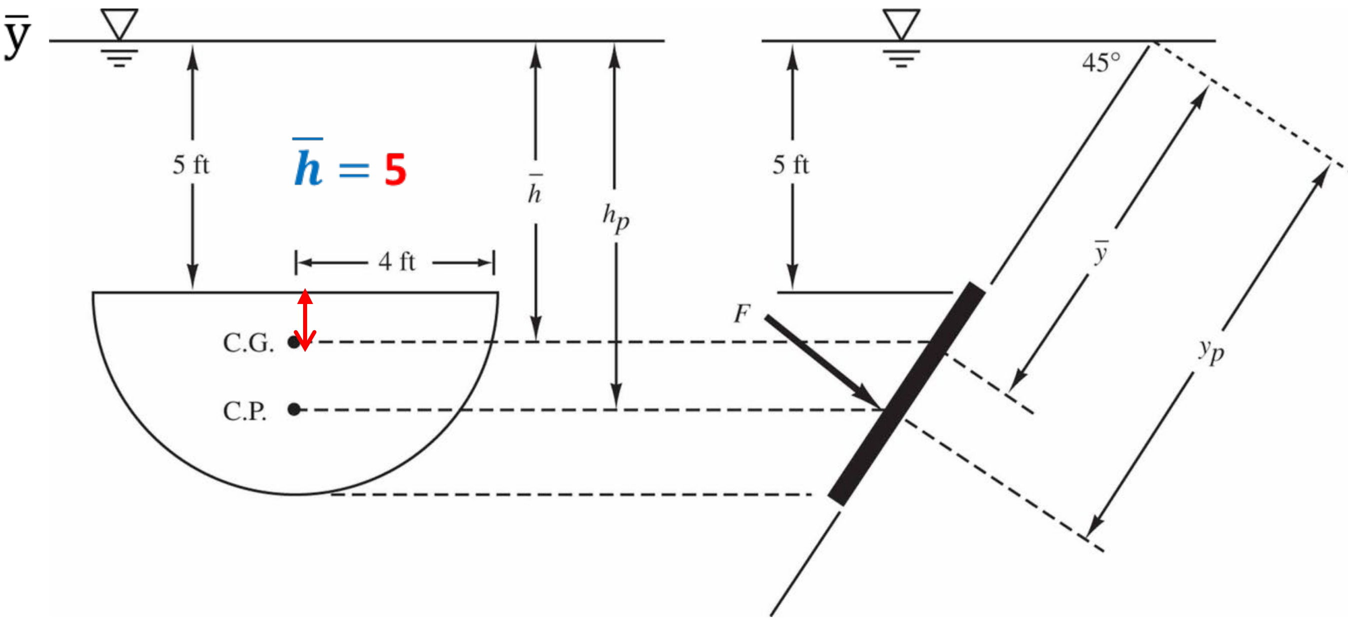
b = 3 m

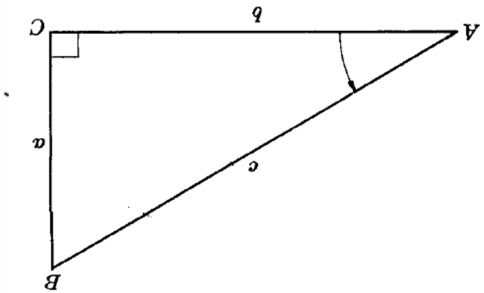
$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{1.22}{4x5.83} + 5.83 = 5.88\ \text{m}$$

$$\overline{h} = h + \overline{y}$$

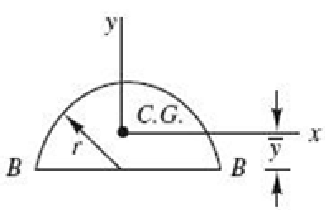
Note: *since the gate 5 m below the free surface, total submergence depth is taken into consideration*

- **Example 15.2 (Hwang et al., 4th Edition):** An inverted semicircular gate is installed at 45 °C with respect to the free water surface (Figure 15.2).The top of the gate is 5 ft below the water surface in the vertical direction. Determine the hydrostatic force and the center of pressure on the gate.





$$\sec A = \frac{c}{b}$$

Semicircle 	$\frac{1}{2}\pi r^2$	$\bar{y} = \frac{4r}{3\pi}$	$I_0 = \frac{(9\pi^2 - 64)r^4}{72\pi}$

$\bar{y} = 1.698\, m$
(Centroid)

$$\bar{y} = 1.698\, m + 5\frac{ft}{Sin(45)} = 8.77\, ft$$

- **Example 15.2 (Hwang et al., 4th Edition):** An inverted semicircular gate is installed at 45 °C with respect to the free water surface (Figure 15.2).The top of the gate is 5 ft below the water surface in the vertical direction. Determine the hydrostatic force and the center of pressure on the gate.

$$\bar{y} = 1.698\, m + \frac{5\, ft}{Sin(45)} = 8.77\, ft$$

Total pressure force=$\gamma\bar{y}\sin\theta\, A$

$$F = \left(62.3\ \frac{\text{Ib}}{\text{ft}^3}\right)(8.77\ \text{ft})(\sin 45)(25.1\ \text{ft}^2) = 9700\ \text{Ibs}$$

$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{28.1}{25.1x8.77}$ +8.77 =8.90 ft (inclined distance from the water surface to the center of pressure

$$I_0 = \frac{(9\pi^2 - 64)r^4}{72\ \pi} = 28.1\ \text{ft}^4$$

PROCEDURE FOR COMPUTING THE FORCE ON A SUBMERGED PLANE AREA

1. Identify the point where the angle of inclination of the area of interest intersects the level of the free surface of the fluid.
2. Locate the centroid of the area from its geometry
3. Determine $\bar{h}$ as the vertical distance from the level of the free surface down to the centroid of the area.
4. Determine the $\bar{y}$ as the inclined distance from the level of the free surface down to the centroid of the area. $\bar{h} = \bar{y} \sin\theta$
5. Calculate the total area A on which the force is to be determined
6. Calculate the resultant force from, $F_R = \gamma \bar{h} A$
7. Calculate I_o, moment of inertia of the area about its centroidal axis.
8. calculate the location of the center of pressure,

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y}$$

4.15 A vat has a sloped side, as shown in Fig. 4.27. Compute the resultant force on this side if the vat contains 4.7 m of glycerine. Also compute the location of the center of pressure and show it on a sketch with the resultant force.

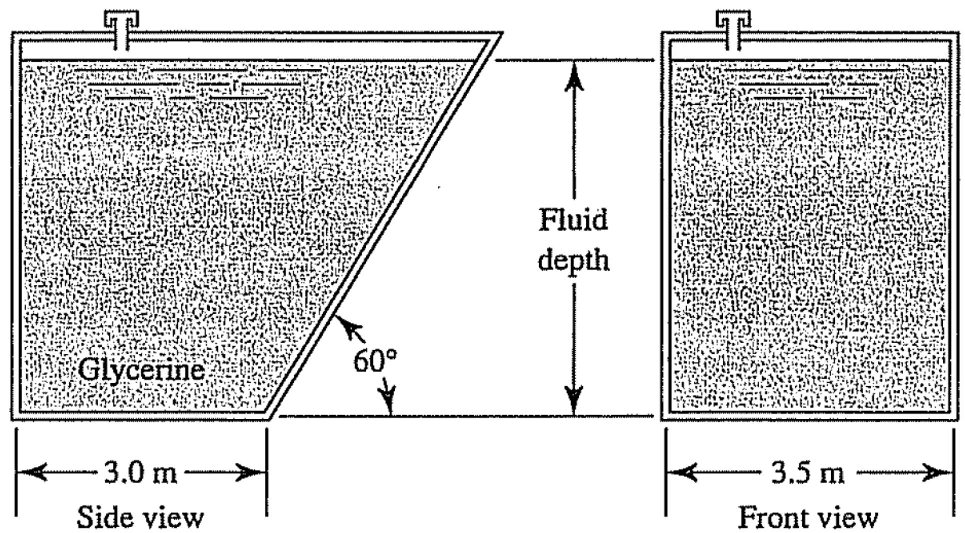


FIGURE 4.27 Vat for Problem 4.15.

4.16 The wall shown in Fig. 4.28 is 6.1 m long. (a) Calculate the total force on the wall due to water pressure and locate the center of pressure; (b) calculate the moment due to this force at the base of the wall.

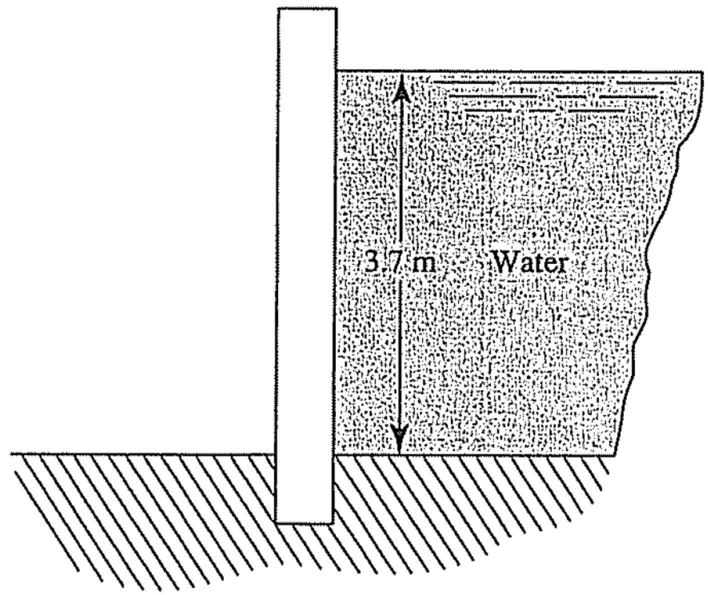
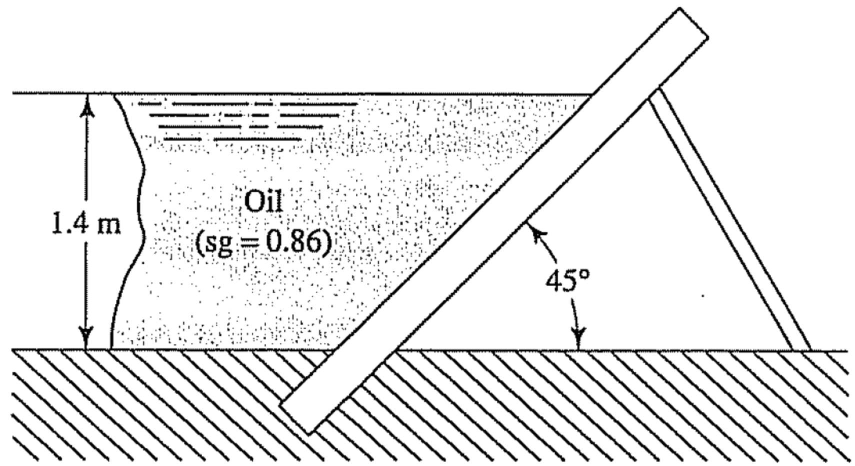


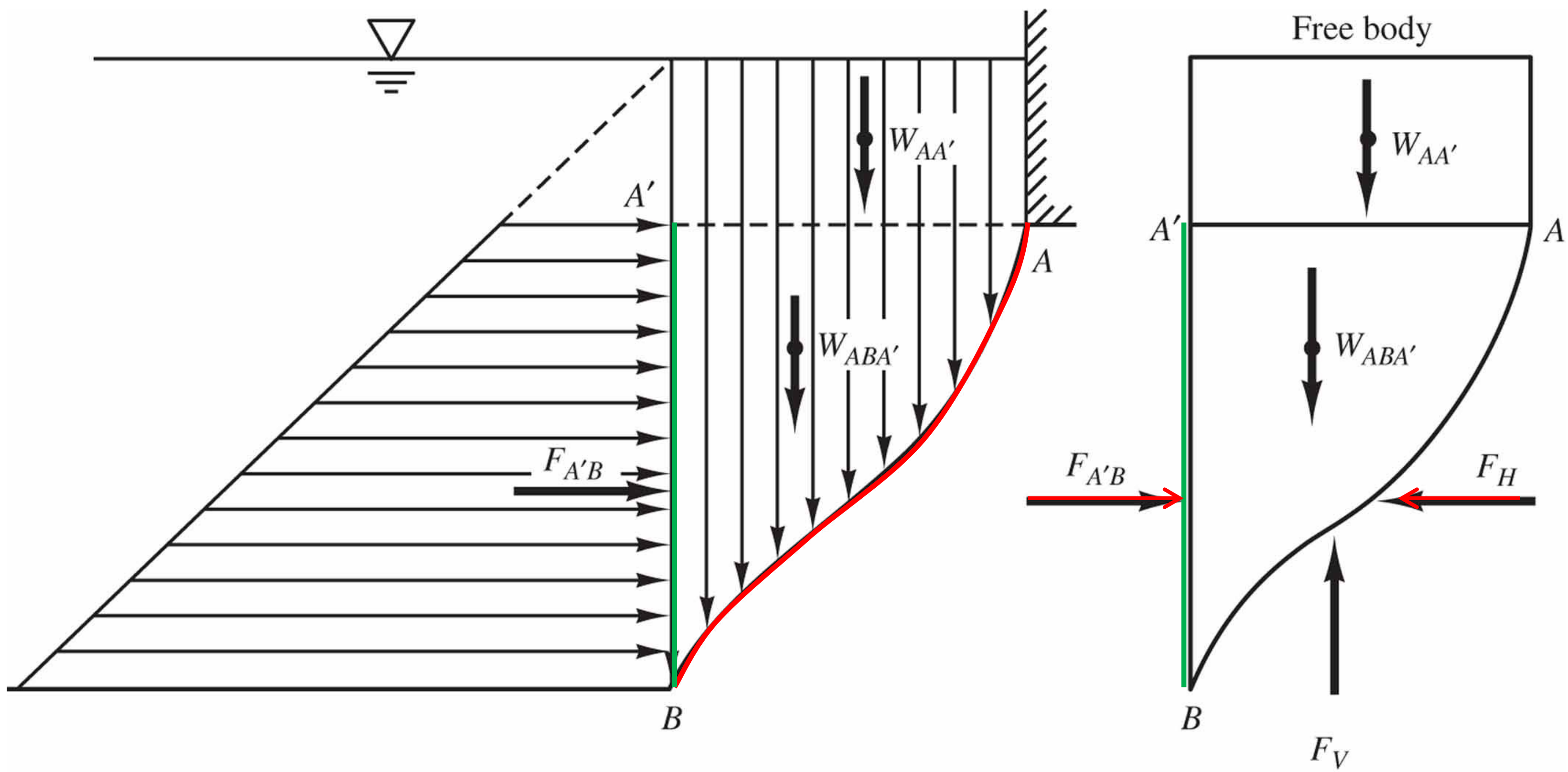
FIGURE 4.28 Problem 4.16.

4.17 If the wall in Fig. 4.29 is 4 m long, calculate the total force on the wall due to the oil pressure. Also determine the location of the center of pressure and show the resultant force on the wall.

FIGURE 4.29 Problem 4.17.

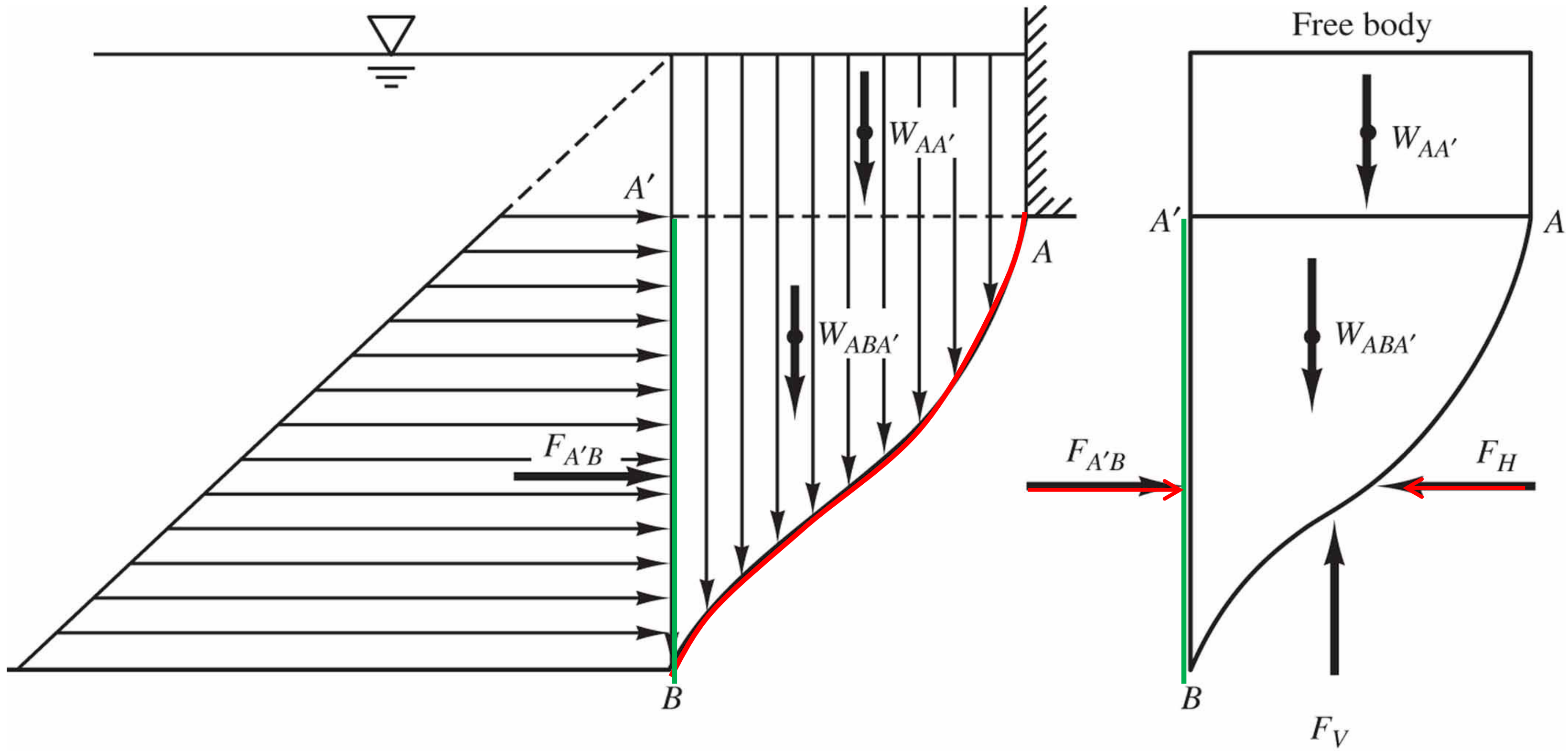


HYDROSTATIC FORCES ON CURVED SURFACES



The water body in the container is stationary, each of the force components must the satisfy the equilibrium conditions➔ $\sum F_X = 0, \sum F_y = 0$

HYDROSTATIC FORCES ON CURVED SURFACES

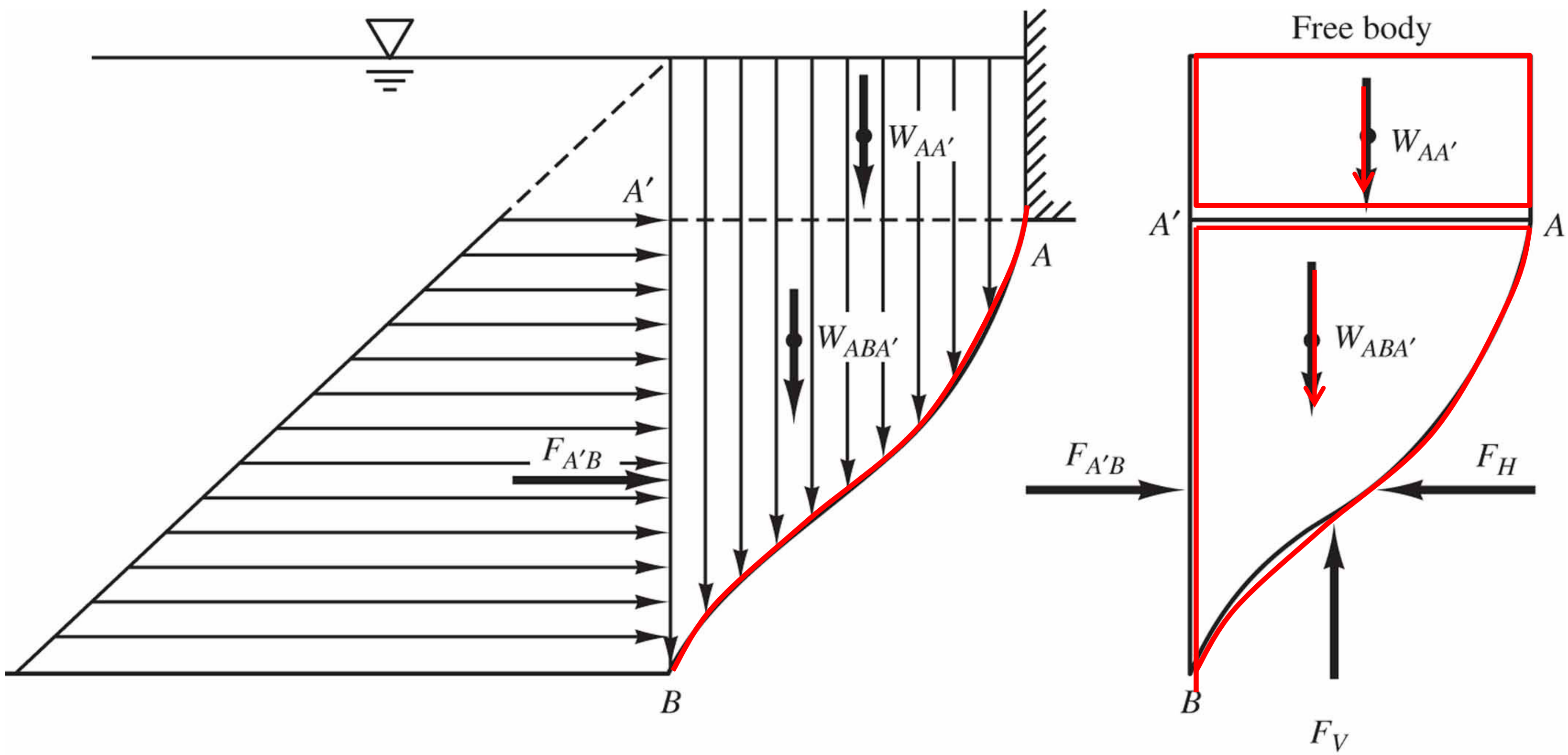


Horizontal forces: $F_{A'B}$ & F_H

$F_{A'B}$: Horizontal pressure exerted on the plane surface A'B

F_H : The force that gate wall exerts on the fluid , $\sum F_X = 0 = F_{A'B} - F_H$ $F_{A'B} = F_H$

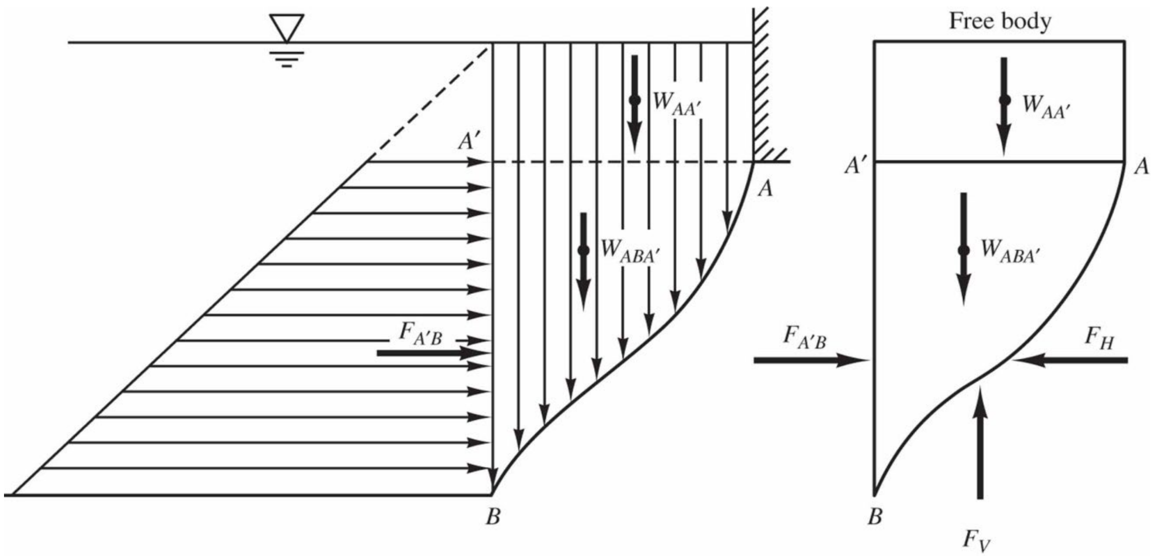
HYDROSTATIC FORCES ON CURVED SURFACES



$$\sum F_y = F_V - (W_{AA'} + W_{ABA'}) = 0$$

$$F_V = W_{AA'} + W_{ABA'}$$

HYDROSTATIC FORCES ON CURVED SURFACES



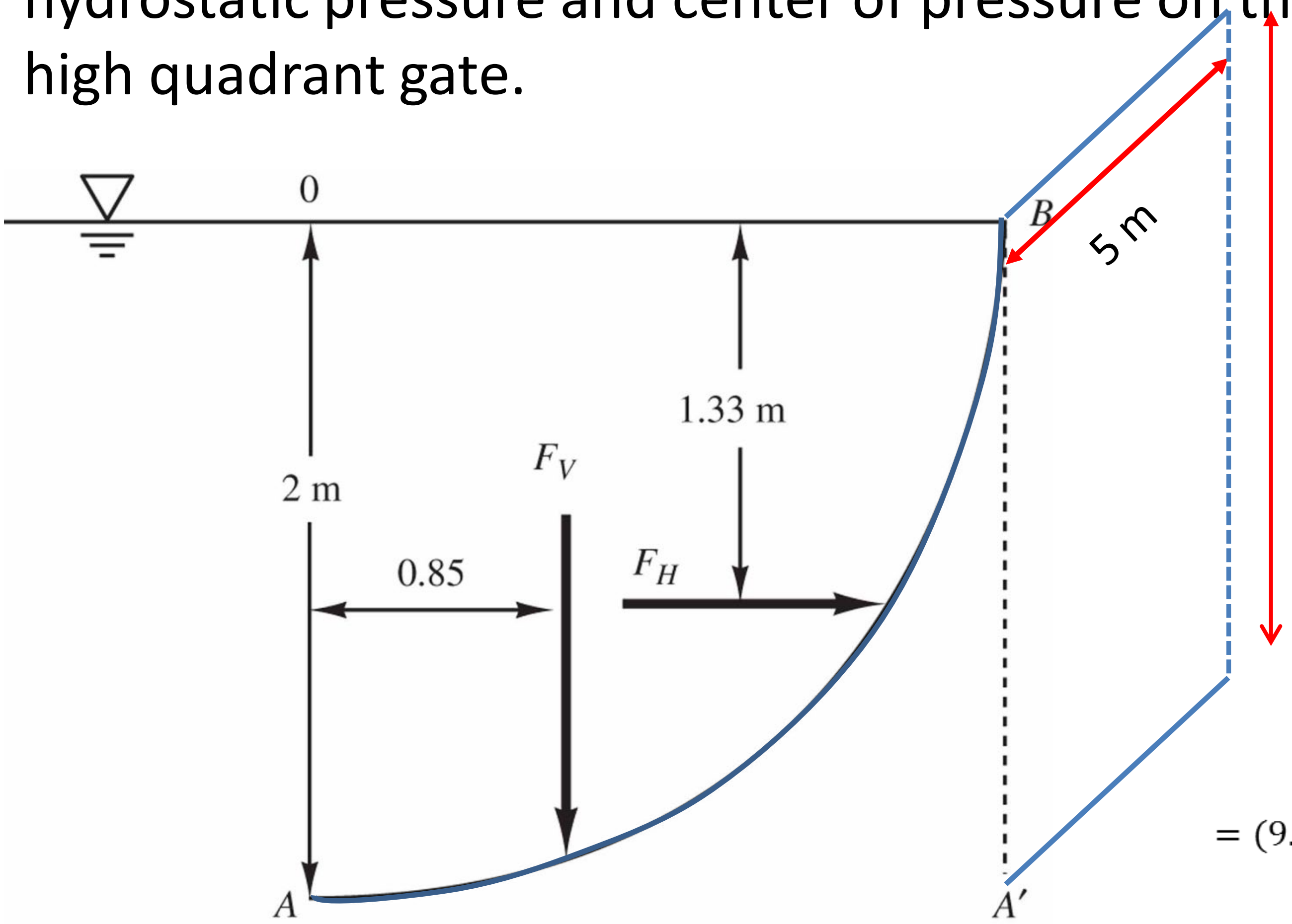
$$F_R = \sqrt{{F_H}^2 + {F_V}^2}$$

$$\text{direction} = \tan^{-1}\left(\frac{F_V}{F_H}\right)$$

1. The horizontal component of the total hydrostatic pressure force on any surface is always equal to the total pressure on the vertical projection of the surface. The resultant force of the horizontal component can be located through the center of pressure of this projection.

2. The vertical component of the total hydrostatic pressure force on any surface is always equal to the weight of the entire water column above the surface extending vertically to the free surface. The resultant force of the vertical component can be located through centroid of this column.

Example 2.5 (Hwang et al., 4th Edition): Determine the total hydrostatic pressure and center of pressure on the 5 m long, 2 m high quadrant gate.



Area of the projected plane : 2 x 5 = 10m^2

Center of gravity of the projected plane (rectangular):

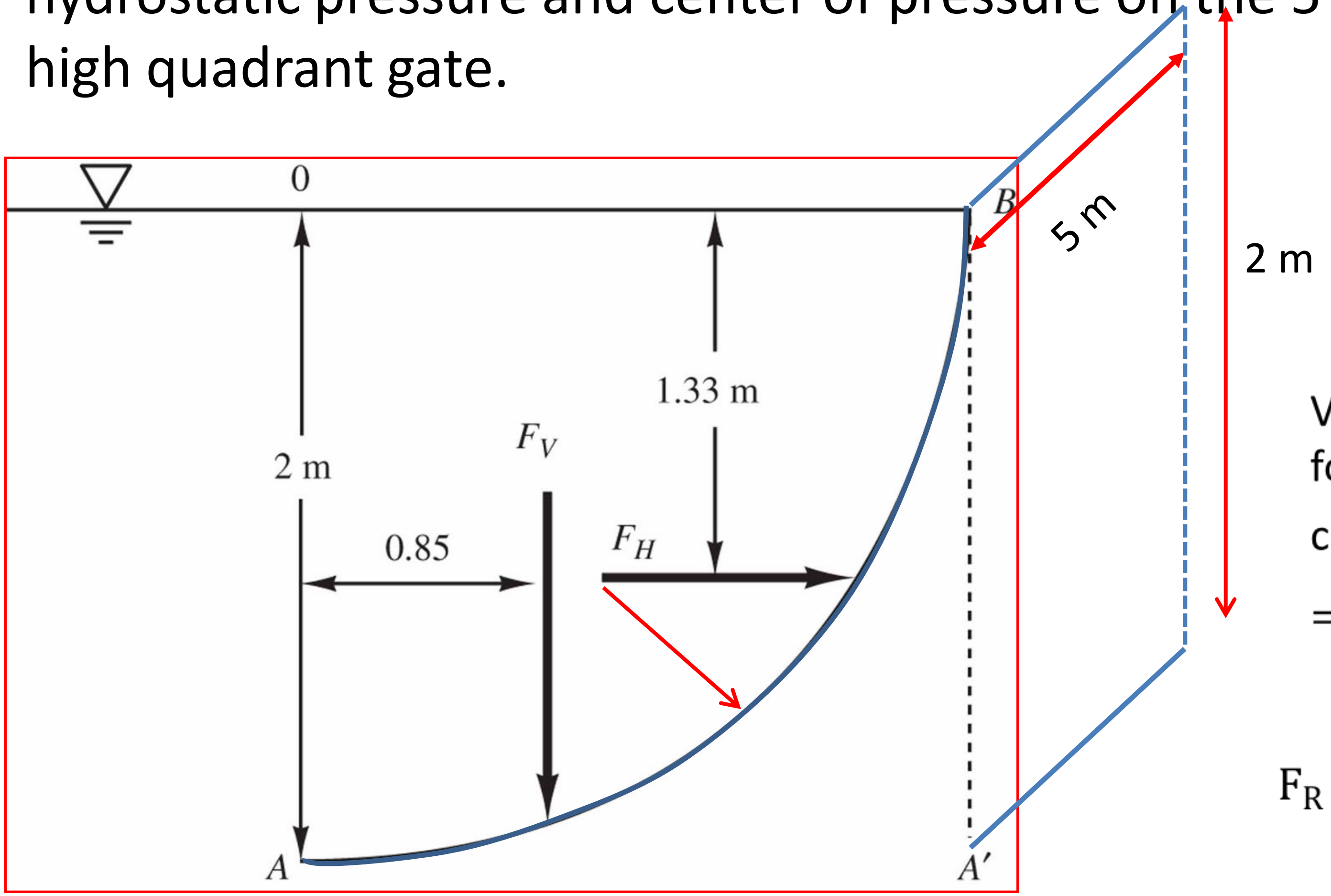
$\bar{y}$= ½ x 2 = 1 m

$\bar{y} = \bar{h}$

$$F_H = \gamma \bar{h} A$$
$$= (9.79\ \mathrm{kN/m^3})(1\ \mathrm{m})(10\ \mathrm{m^2})$$
$$= 97.9\ \mathrm{kN}$$

Location of the F_H ➔ $y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{3.33}{10(1)} + 1 = 1.33\ \mathrm{m}$

Example 2.5 (Hwang et al., 4th Edition): Determine the total hydrostatic pressure and center of pressure on the 5 m long, 2 m high quadrant gate.



Vertical pressure force located at the centroid. $\bar{y} = \frac{4r}{3\pi} = \frac{4(2\text{ m})}{3(3.14)} = 0.85\text{ m}$

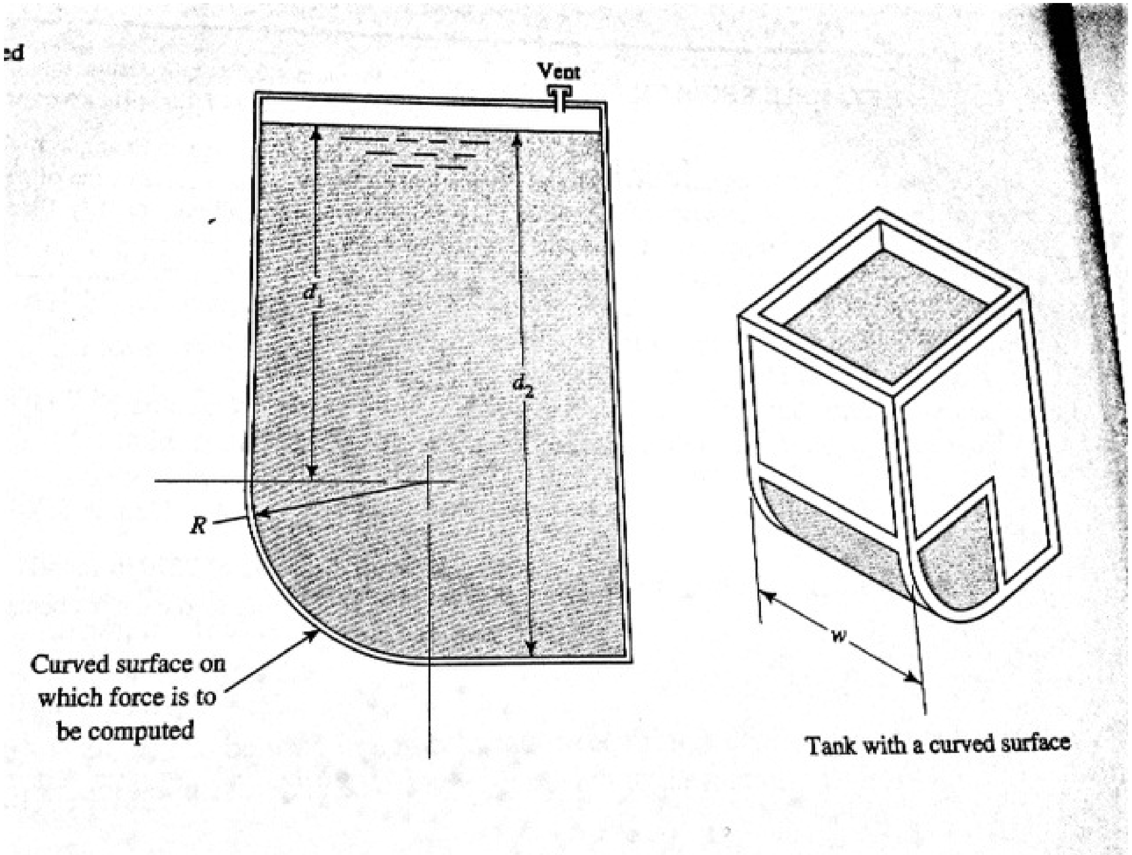
$$F_R = \sqrt{F^2{}_H + F^2{}_V} = \sqrt{97.9^2 + 154^2} = 182\text{ kN}$$

Vertical component = weight of the water in the volume AOB

$$F_V = \gamma\forall = \gamma Ah = (9.79)\left(\frac{1}{4}\pi(2\text{ m})^2\right)(5\text{ m}) = 154\text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right) = \left(\frac{154}{97.9}\right) \Rightarrow 57.6°$$

Example. Compute the horizontal and vertical components of the resultant force on the curved surface and the resultant force itself. Show these force vectors on a sketch.



d1= 3.0 m

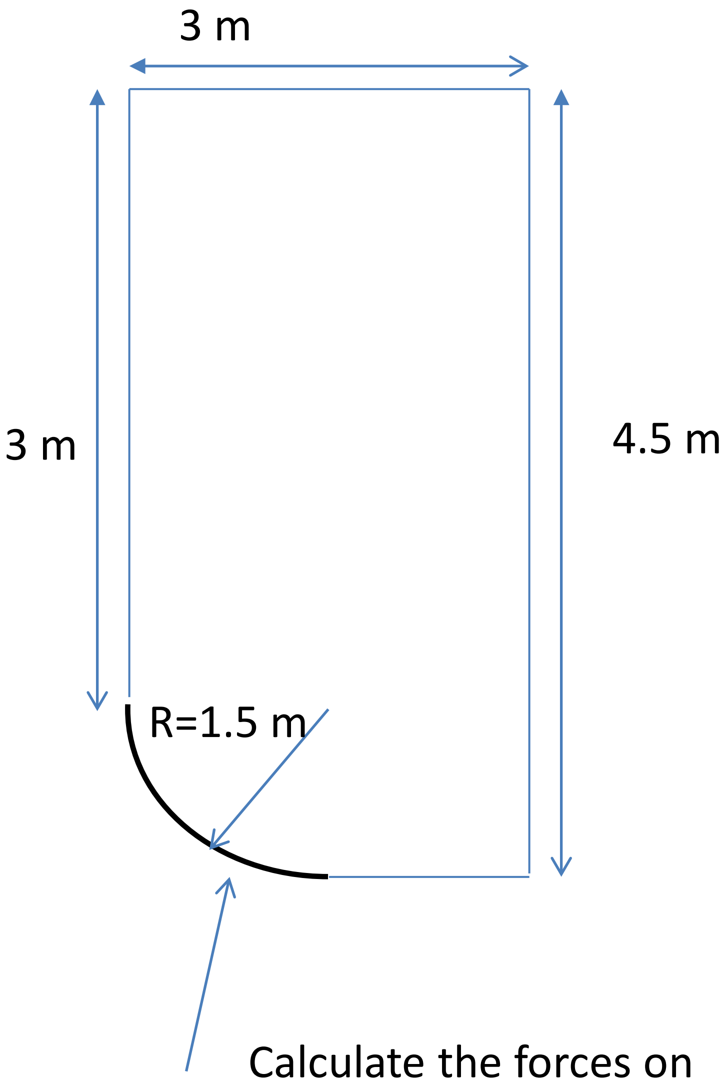
d2 = 4.50 m

w=2.5 m

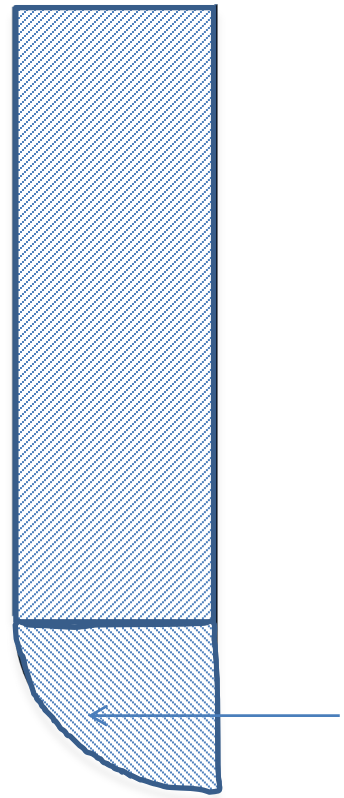
R= 1.5 m

Liquid is water

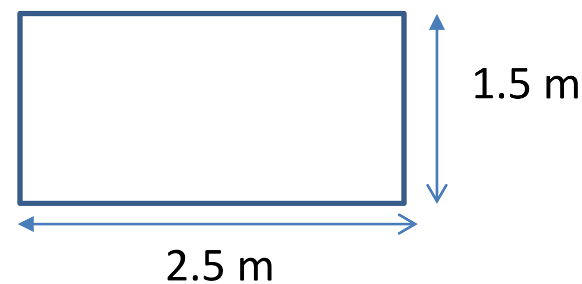
HYDROSTATIC FORCES ON CURVED SURFACE



Calculate the forces on curved surface

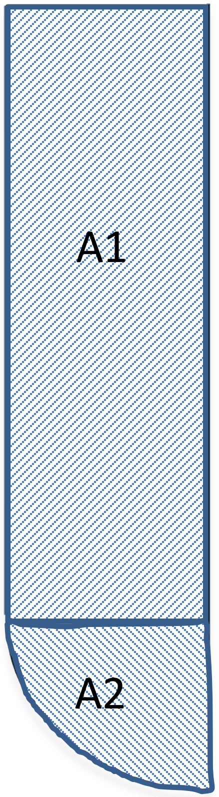


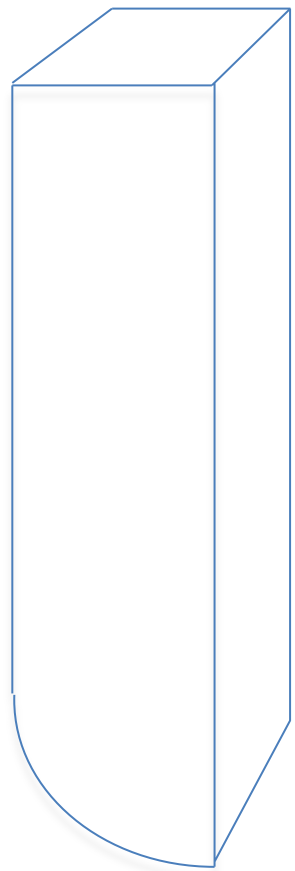
Isolated volume above the curved surface



Vertical projection of the curved surface

HYDROSTATIC FORCES ON CURVED SURFACE





Volume = Area ×h=6.267 m^2×2.5m

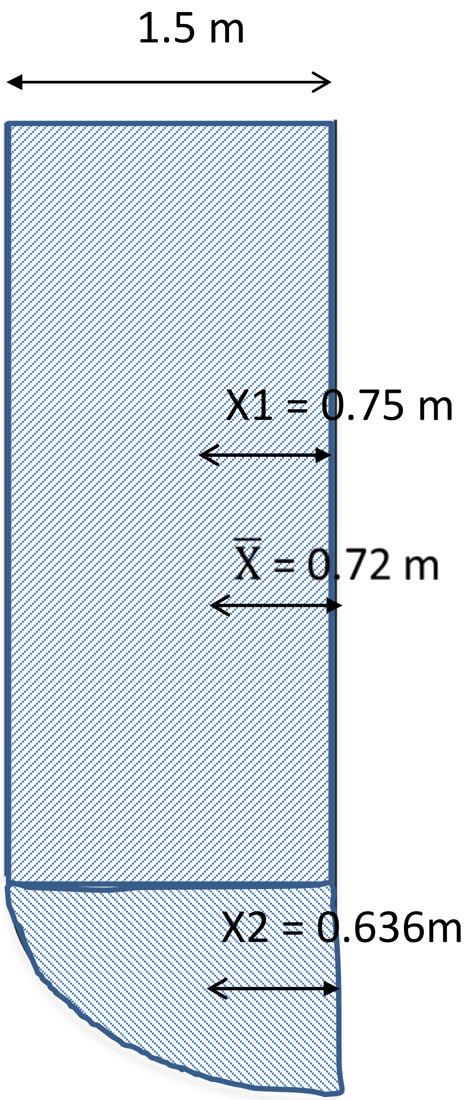
Weight = $\gamma \times \forall = (9.81) \times (15.67\ \mathrm{m}^3) = 153.7\ \mathrm{kN}$

Fv is acting downward through the centroid of the volume

Centroid of the volume is found by composite area technique

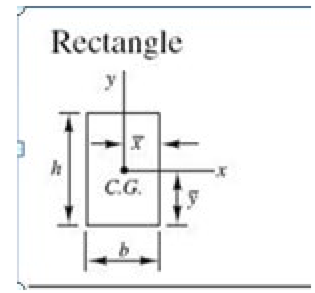
Area = $A_1 + A_2 = (3\ \mathrm{m} x 1.5) + (\frac{1}{4} \times \pi \times (1.5)^2\ )$

HYDROSTATIC FORCES ON CURVED SURFACE



Centroid of the area A1 ➔

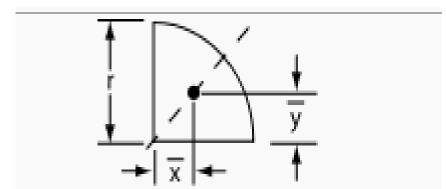
$X_1 = \frac{1}{2} b = \frac{1}{2}(1.5 \text{ m}) = 0.75 \text{ m}$



Centroid of the area A2 ➔

$X_2 = \frac{4}{3}\frac{r}{\pi} = \frac{4}{3}\frac{(1.5)}{(3.14)} = 0.64 \text{ m}$

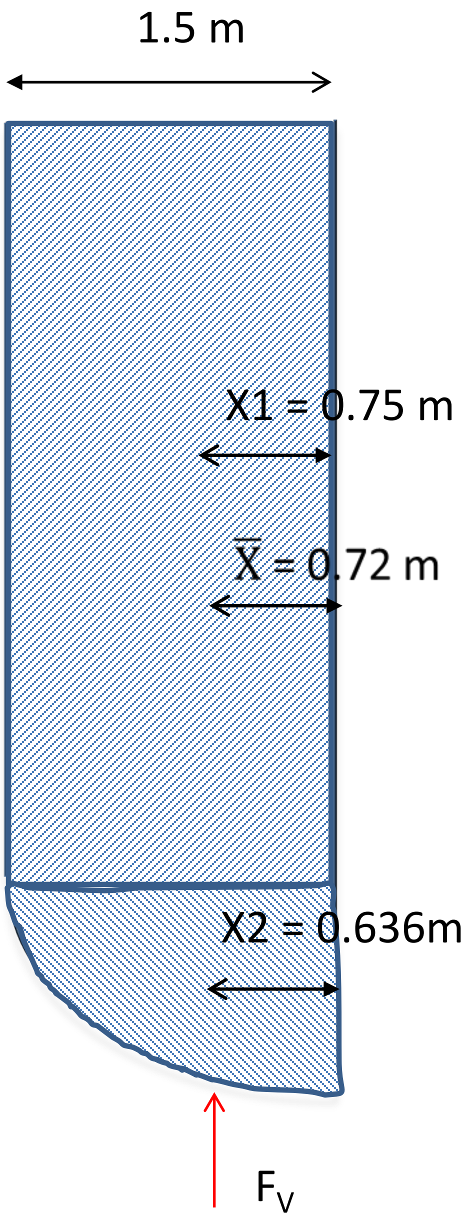
Centroid of the quadrant

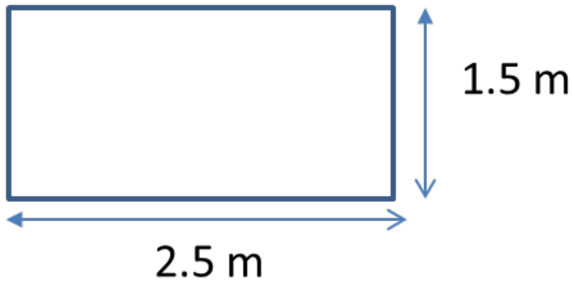


$\overline{X} = \frac{A_1 \times X_1 + A_2 \times X_2}{A_1 + A_2} = \frac{(4.5)(0.75)+(1.767)(0.64)}{4.5+1.767}$

$= 0.72 \text{ m}$

HYDROSTATIC FORCES ON CURVED SURFACE





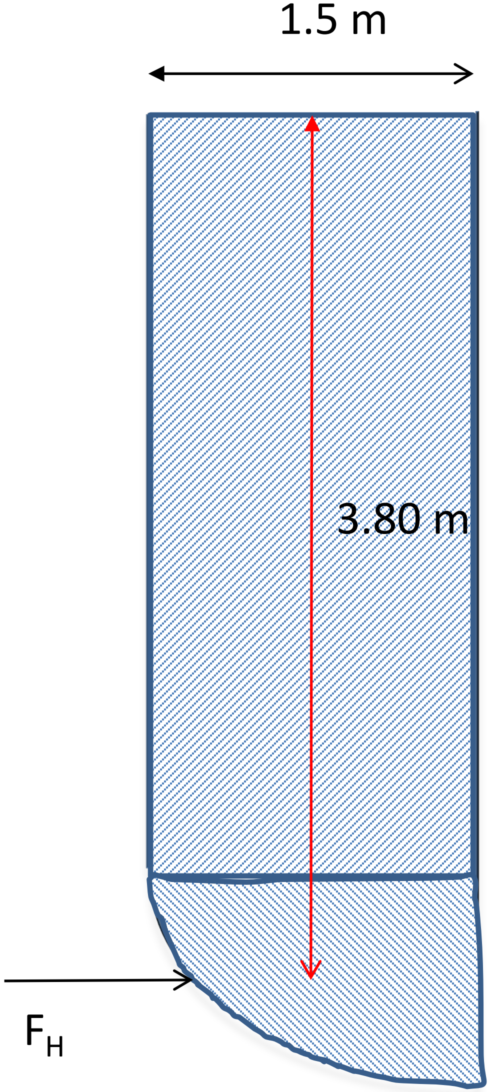
Vertical projection of the curved surface

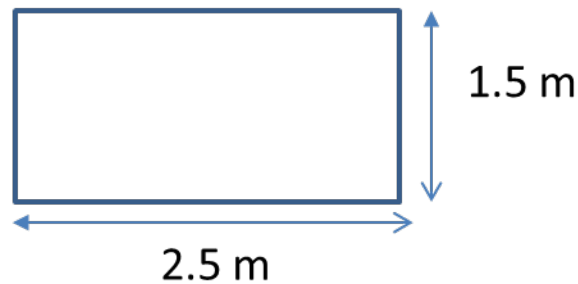
Depth to the centroid of the projected area

$$\bar{h} = 3 + \frac{1.5\ m}{2} = 3.75\ \text{m}$$

$$F_H = \gamma(Area)\bar{h} = (9.81)(2.5)(1.5)(3.75m) = 138\ \text{kN}$$

HYDROSTATIC FORCES ON CURVED SURFACE





Vertical projection of the curved surface

$$F_H = \gamma(Area)\overline{h} = (9.81)(2.5)(1.5)(3.75m) = 138 \text{ kN}$$

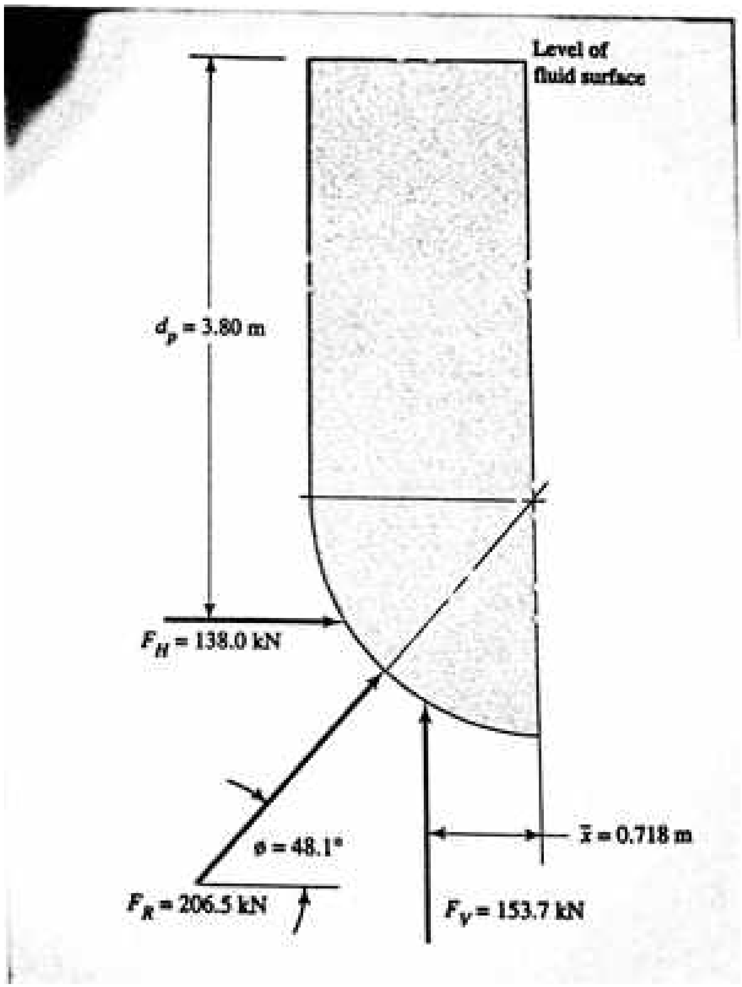
The depth of the line of action of the horizontal compound

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y}$$

$$I_0 = \frac{1}{12}bh^3 = \frac{1}{12}(2.5)(1.5)^3$$

$$y_P = \frac{\left(\frac{1}{12}\right)\times(2.5)\times(1.5)^3}{(2.5)\times(1.5)\times(3.75)} + 3.75 = 3.80 \text{ m}$$

HYDROSTATIC FORCES ON CURVED SURFACE



$$F_R = \sqrt{F^2{}_V + F^2{}_H}$$

$$= \sqrt{(153.7)^2 + (138)^2} \quad = 206.5 \text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right) = \tan^{-1}\left(\frac{153.7}{138}\right) = 48.1°$$