ENVE 2061 BASIC FLUID MECHANICS

Fluid Statics:

Pressure At a Point

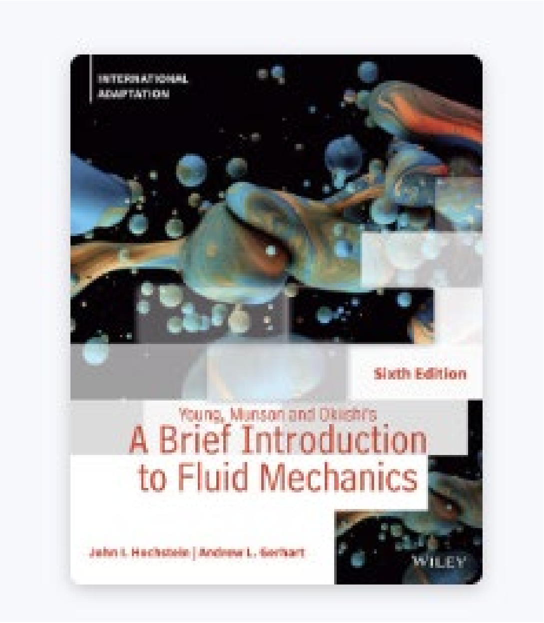
Basic Equation for Pressure Field

Change of Pressure in Static Liquid

Assoc. Prof. Neslihan Semerci

Lecture Notes from

- A Brief Introduction to Fluid Mechanics, 6th Edition, Donald F. Young, Bruce R. Munson, Theodore H. Okiishi, Wade W. Huebsch



Learning Objectives:

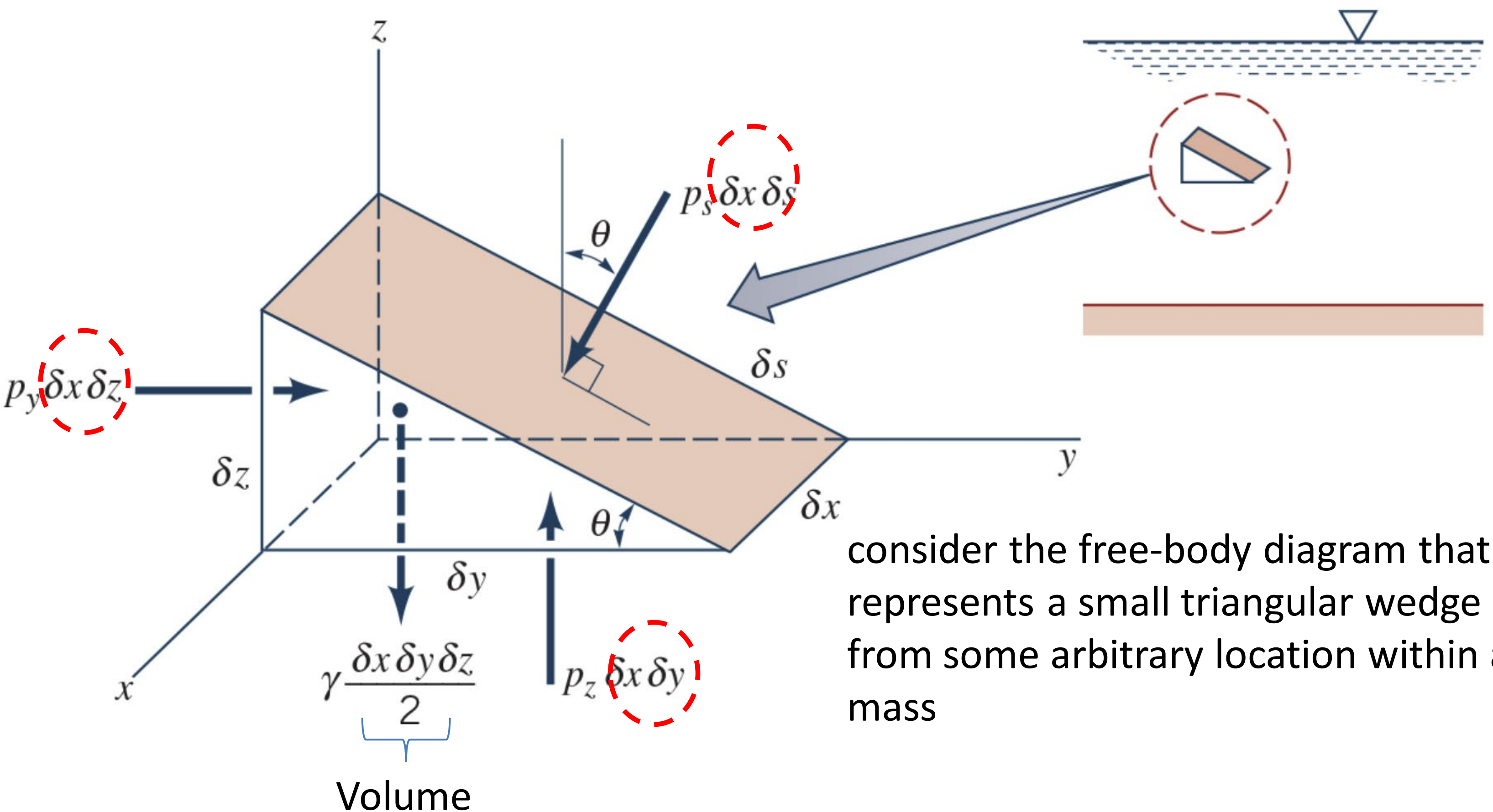
- determine the pressure at various locations in a fluid at rest.

Pressure at a point

Question: how the pressure at a point varies with the orientation of the plane passing through the point?

$$\sum F_y = p_y\,\delta x\,\delta z - p_s\,\delta x\,\delta s \sin\theta = \rho\frac{\delta x\,\delta y\,\delta z}{2}a_y$$

$$\sum F_z = p_z\,\delta x\,\delta y - p_s\,\delta x\,\delta s \cos\theta - \gamma\frac{\delta x\,\delta y\,\delta z}{2} = \rho\frac{\delta x\,\delta y\,\delta z}{2}a_z$$



consider the free-body diagram that represents a small triangular wedge of fluid from some arbitrary location within a fluid mass

Pressure at a point

Since we are considering the situation in which there are no shearing stresses, the only external forces acting on the wedge are due to **the pressure and gravity.**

Shearing stresses is zero; as the fluid mass moves as a rigid body; that is, there is no relative motion between adjacent elements.

$$\sum F_y = p_y\,\delta x\,\delta z - p_s\,\delta x\,\delta s \sin\theta = \rho\frac{\delta x\,\delta y\,\delta z}{2}a_y$$

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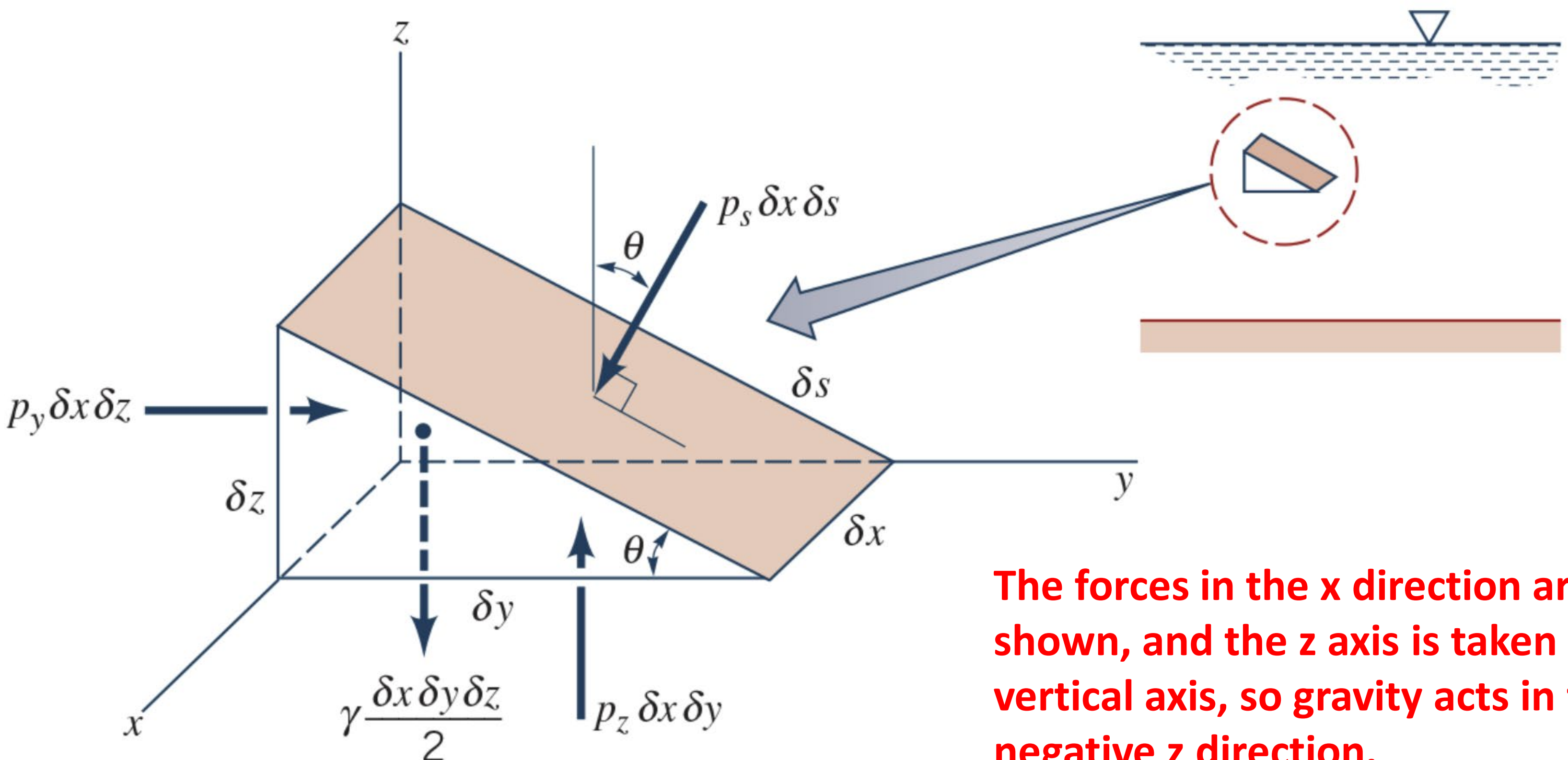

The forces in the x direction are not shown, and the z axis is taken as the vertical axis, so gravity acts in the negative z direction.

Pressure at a point

Apply Newton’s second law; **F=m.a**

$$\sum F_y = p_y\,\delta x\,\delta z - p_s\,\delta x\,\delta s\sin\theta = \rho\frac{\delta x\,\delta y\,\delta z}{2}a_y$$

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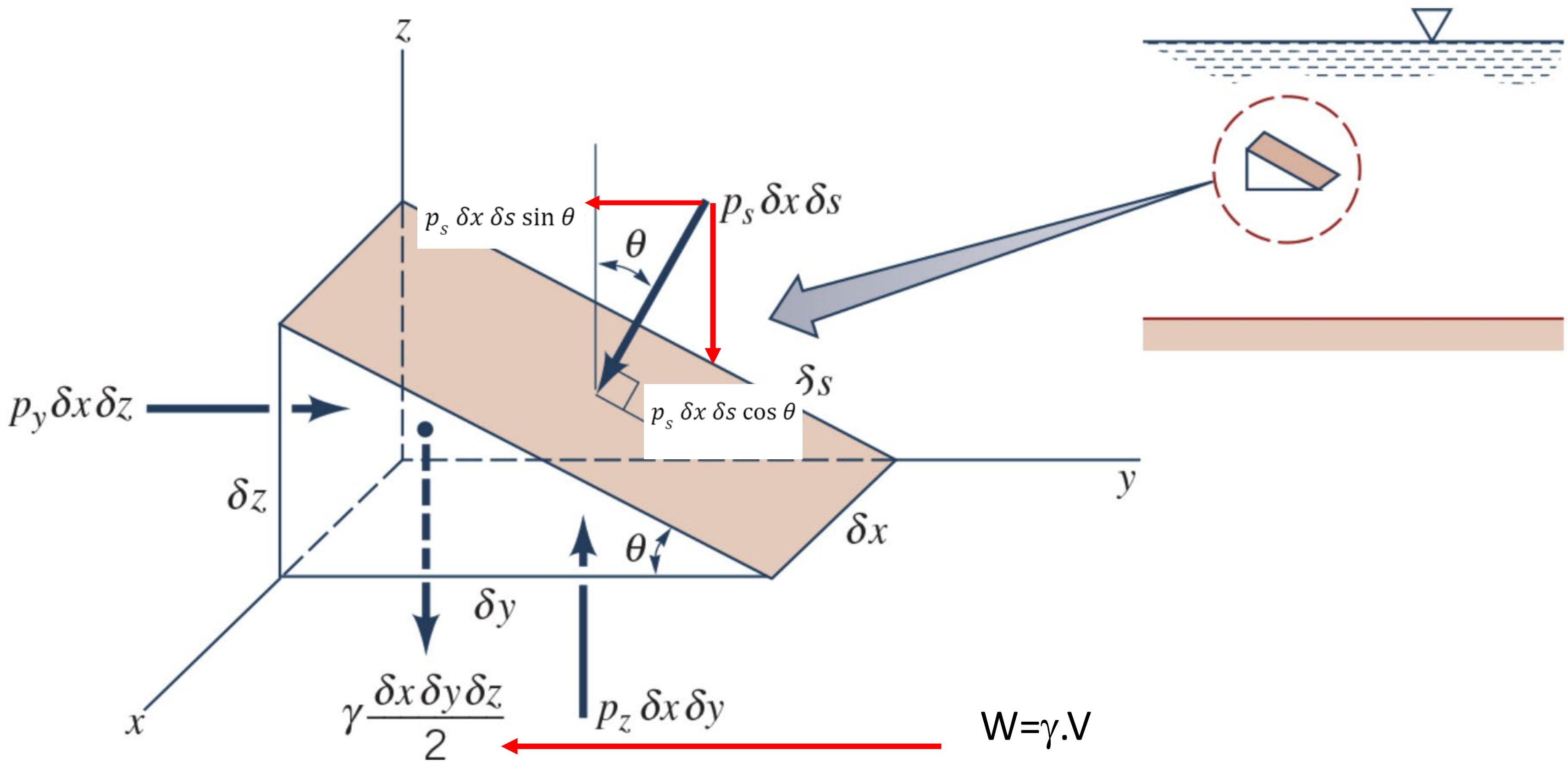

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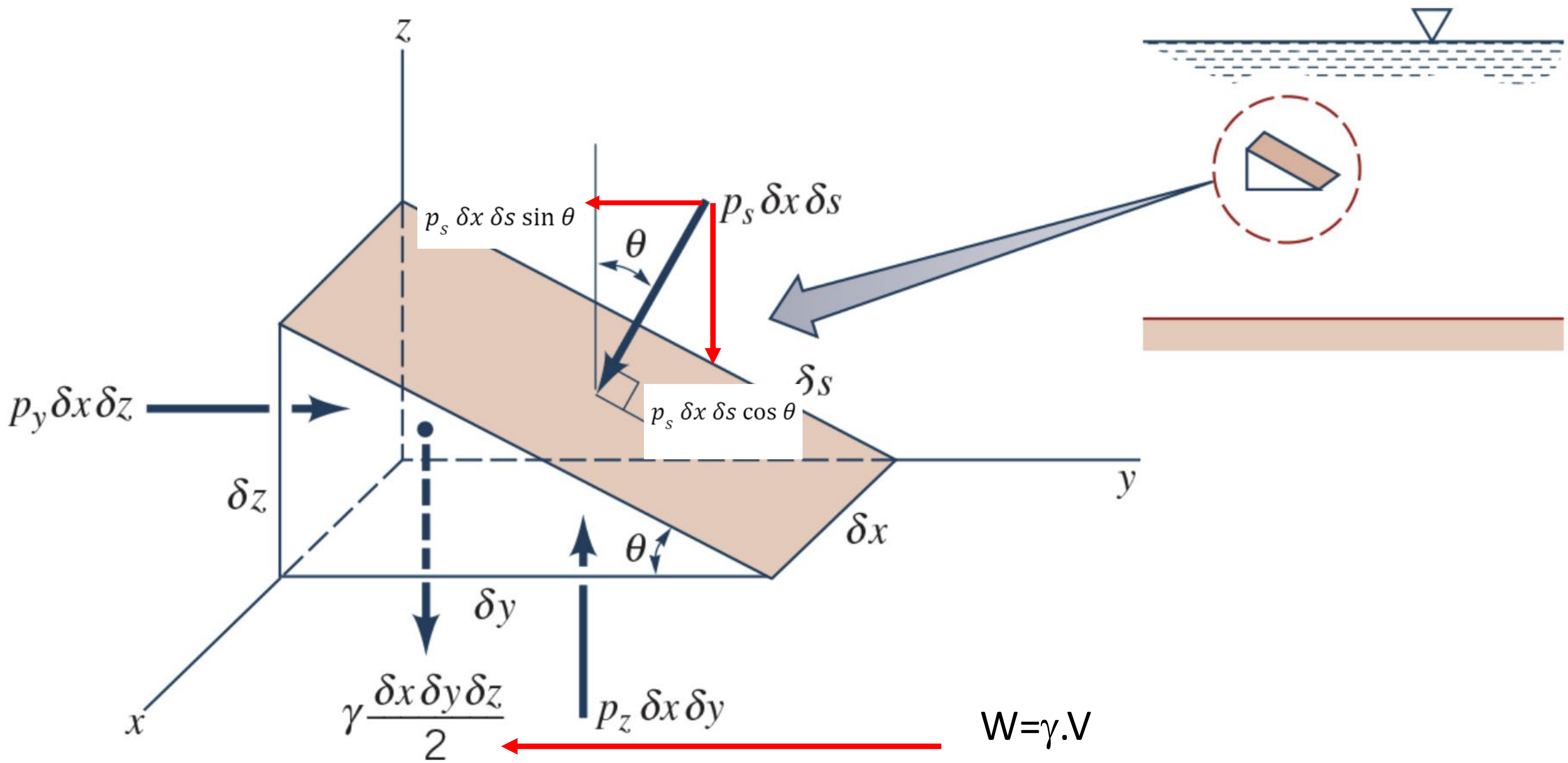

Pressure at a point

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- Average pressures on the faces: ps, py, and pz
- Fluid Specific Weight: γ
- Density: ρ
- Accelerations: ay and az

Note: that a pressure must be multiplied by an appropriate area to obtain the force due to the pressure. It follows from the geometry that

$$\delta y = \delta s\ \cos\ \theta \qquad \delta z = \delta s\ \sin\ \theta$$

$$p_y - p_s = \rho a_y\,\frac{\delta y}{2}$$

$$p_z - p_s = (\rho a_z + \gamma)\,\frac{\delta z}{2}$$

Since we are really interested in what is happening at a point, we take the limit as δx, δy, and δz approach zero (while maintaining the angle θ), and it follows that

Pressure at a point

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Since we are really interested in what is happening at a point, we take the limit as δx, δy, and δz approach zero (while maintaining the angle θ), and it follows that

$$p_y = p_s \qquad p_z = p_s$$

ps = py = pz

Conclusion: ***pressure at a point in a fluid at rest, or in motion, is independent of direction as long as there are no shearing stresses present*. Pascal's law**

Pascal's law, named in honor of Blaise Pascal (1623–1662), a French mathematician who made important contributions in the field of hydrostatics.

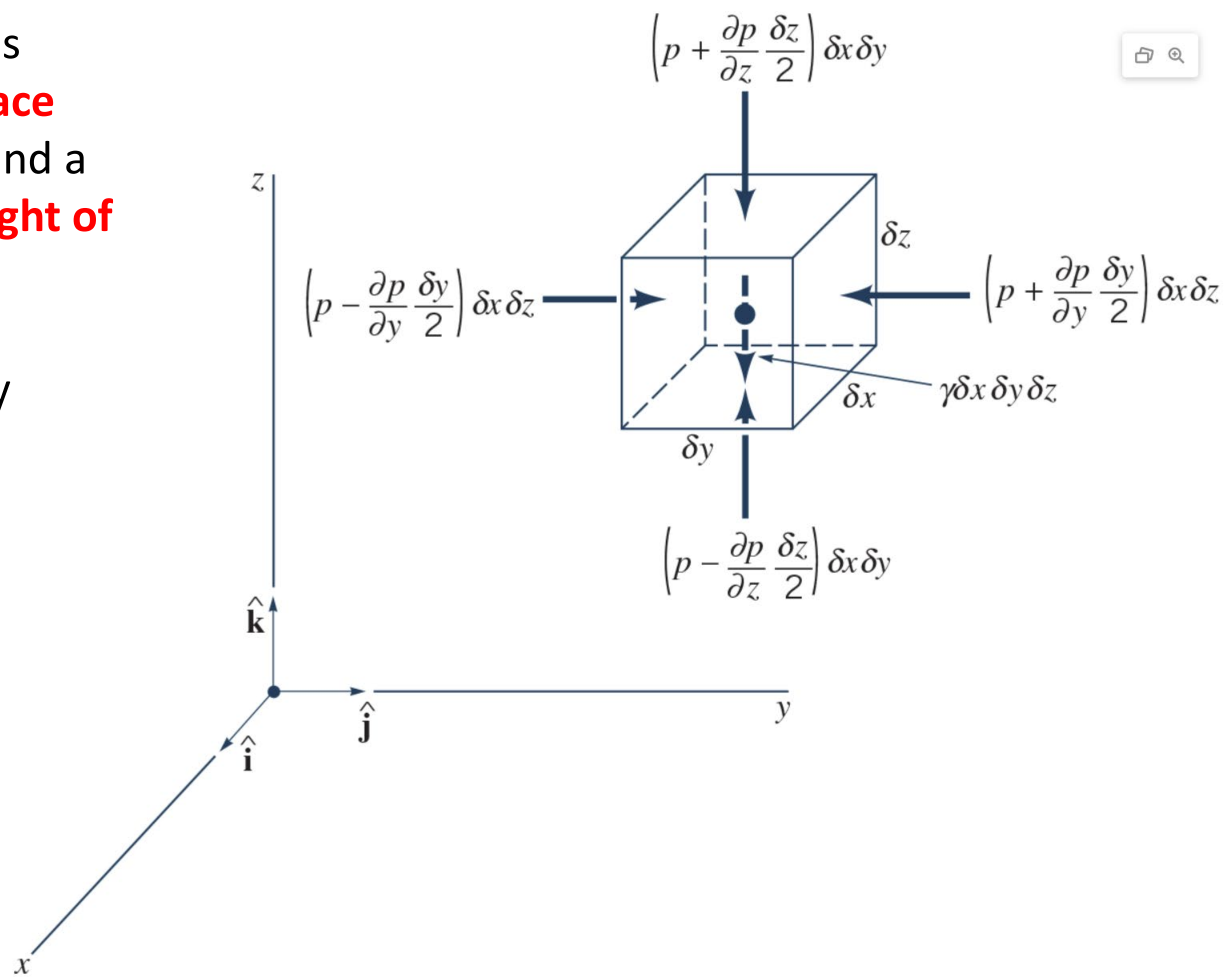
BASIC EQUATION FOR PRESSURE FIELD

2nd Question: how does the pressure in a fluid in which there are no shearing stresses vary from point to point?

consider a small rectangular element of fluid removed from some arbitrary position within the mass of fluid of interest

There are two types of forces acting on this element: **surface forces due to the pressure** and a **body force equal to the weight of the element**.

Other possible types of body forces, such as those due to magnetic fields, will not be considered.



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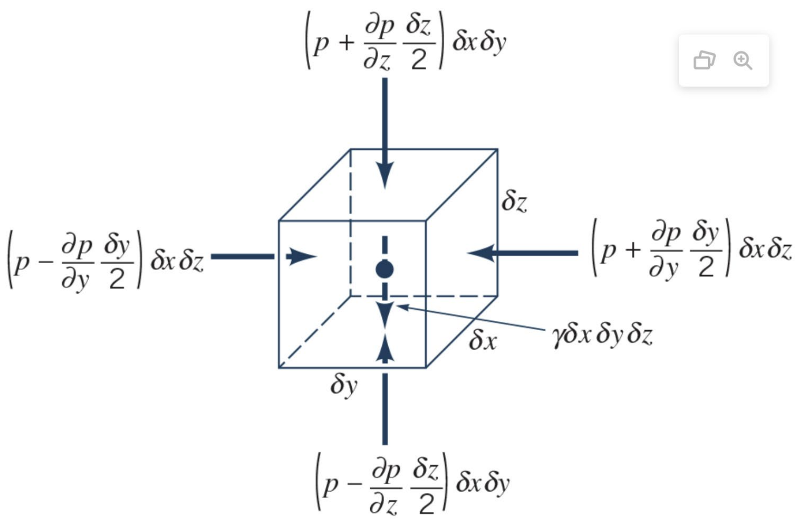


Surface and body forces acting on small fluid element.

The resultant surface force in the y direction is

$$\delta F_y = \left(p - \frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x \delta z - \left(p + \frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x\ \delta z$$

$$\delta F_y = -\frac{\partial p}{\partial y}\delta x\ \delta y\ \delta z$$

The resultant surface forces in the x and direction are

$$\delta F_x = -\frac{\partial p}{\partial x}\ \delta x\ \delta y\ \delta z \quad \delta F_z = -\frac{\partial p}{\partial z}\ \delta x\ \delta y\ \delta z$$

The resultant surface force acting on the element due to pressure can be expressed in vector form as

$$\delta \mathbf{F}_s = \delta F_x \hat{\mathbf{i}} + \delta F_y \hat{\mathbf{j}} + \delta F_z \widehat{\mathbf{k}}$$

$$\delta \mathbf{F}_s = -\left(\frac{\partial p}{\partial x}\hat{\mathbf{i}} + \frac{\partial p}{\partial y}\hat{\mathbf{j}} + \frac{\partial p}{\partial z}\widehat{\mathbf{k}}\right)\ \delta x\ \delta y\ \delta z$$

$\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are the unit vectors along the coordinate axes

$$\frac{\partial p}{\partial x}\hat{\mathbf{i}} + \frac{\partial p}{\partial y}\hat{\mathbf{j}} + \frac{\partial p}{\partial z}\widehat{\mathbf{k}} = \nabla p$$

$$\nabla(\,) = \frac{\partial(\,)}{\partial x}\hat{\mathbf{i}} + \frac{\partial(\,)}{\partial y}\hat{\mathbf{j}} + \frac{\partial(\,)}{\partial z}\widehat{\mathbf{k}}$$

BASIC EQUATION FOR PRESSURE FIELD

$$\delta \mathbf{F}_s = \delta F_x \hat{\mathbf{i}} + \delta F_y \hat{\mathbf{j}} + \delta F_z \widehat{\mathbf{k}}$$

$$\delta F_x = -\frac{\partial p}{\partial x}\, \delta x\, \delta y\, \delta z \quad \delta F_z = -\frac{\partial p}{\partial z}\, \delta x\, \delta y\, \delta z$$

$$\delta \mathbf{F}_s = -\left(\frac{\partial p}{\partial x} \hat{\mathbf{i}} + \frac{\partial p}{\partial y} \hat{\mathbf{j}} + \frac{\partial p}{\partial z} \widehat{\mathbf{k}} \right) \delta x\, \delta y\, \delta z$$

$$\delta F_y = -\frac{\partial p}{\partial y} \delta x\, \delta y\, \delta z$$

$$\frac{\delta \mathbf{F}_s}{\delta x\, \delta y\, \delta z} = -\nabla p$$

The resultant surface force acting on a small fluid element depends only on the pressure gradient if there are no shearing stresses present.

Since the z axis is vertical, the weight of the element is

$$-\delta \mathscr{W} \widehat{\mathbf{k}} = -\gamma\, \delta x\, \delta y\, \delta z\, \hat{\mathbf{k}}$$

the negative sign indicates that the force due to the weight is downward (in the negative z direction).

BASIC EQUATION FOR PRESSURE FIELD

Newton's second law, applied to the fluid element

$$\sum \delta \mathbf{F} = \delta m \mathbf{a}$$

Σ δF represents the **resultant force** acting on the element, **a is the acceleration of the element**, and **δm is the element mass**, which can be written as ρ δx δy δz. It follows that

$$\sum \delta \mathbf{F} = \delta \mathbf{F}_s - \delta \mathscr{W} \hat{\mathbf{k}} = \delta m \mathbf{a}$$

$$\boxed{-\nabla p - \gamma \hat{\mathbf{k}} = \rho \mathbf{a}}$$

general equation of motion for a fluid in which there are no shearing stresses

For a fluid at rest, **a** = 0

$$\nabla p + \gamma \hat{\mathbf{k}} = 0 \qquad \nabla p = -\gamma \hat{\mathbf{k}}$$

$$\frac{\partial p}{\partial x}\hat{\mathbf{i}} + \frac{\partial p}{\partial y}\hat{\mathbf{j}} + \frac{\partial p}{\partial z}\hat{\mathbf{k}} = \nabla p = -\gamma \hat{\mathbf{k}}$$

$$\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = 0 \qquad \frac{\partial p}{\partial z} = -\gamma$$

BASIC EQUATION FOR PRESSURE FIELD

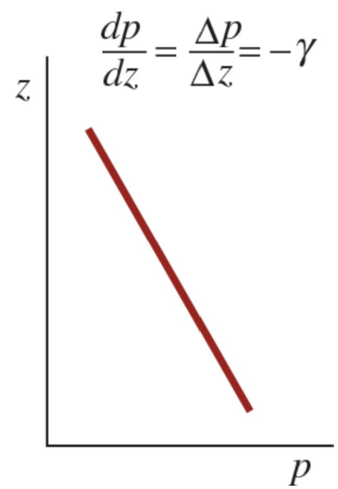
$$\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = 0$$

we move from point to point in a horizontal plane (any plane parallel to the x–y plane), the pressure does not change.

$$\frac{\partial p}{\partial z} = -\gamma \longrightarrow \boxed{\frac{dp}{dz} = -\gamma}$$

can be written as the ordinary differential equation, since p depends only on z,

For liquids or gases at rest, the pressure gradient in the vertical direction at any point in a fluid depends only on the specific weight of the fluid at that point.



CHANGE OF PRESSURE: INCOMPRESSIBLE FLUIDS

$$\boxed{\frac{dp}{dz} = -\gamma}$$

$$\gamma = \rho g$$

most engineering applications the variation in g is negligible

In general, a fluid with constant density is called an **incompressible fluid**.

$$\int_{p_1}^{p_2} dp = -\gamma \int_{z_1}^{z_2} dz$$

$$p_2 - p_1 = -\gamma(z_2 - z_1)$$

h is the distance, $z_2 - z_1$, which is the depth of fluid measured downward from the location of P_2

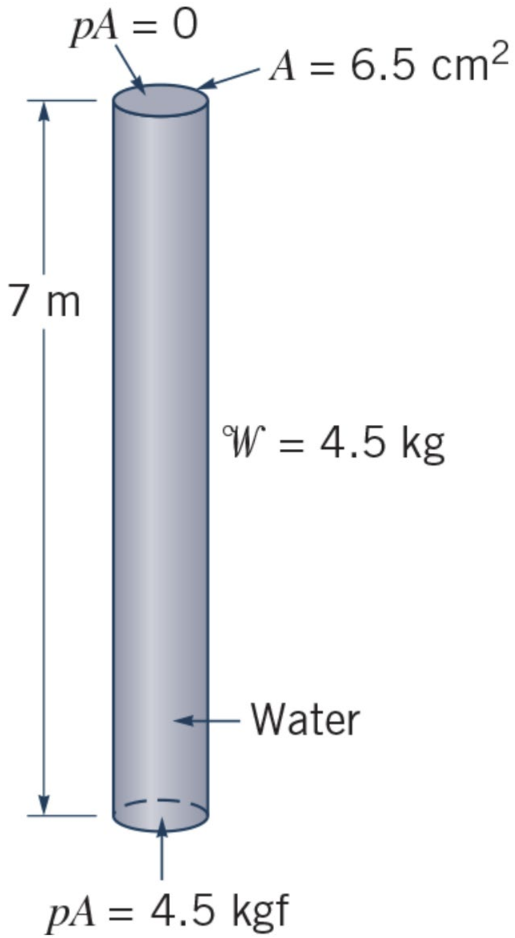
$$p_1 - p_2 = \gamma\underbrace{(z_2 - z_1)}_{h}$$

h is called the pressure head and is interpreted as the height of a column of fluid of specific weight γ required to give a pressure difference p1 − p2.

$$p_1 - p_2 = \gamma h$$

the pressure difference between two points can be specified by the distance h,

CHANGE OF PRESSURE: INCOMPRESSIBLE FLUIDS



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Pressure difference = 69 kPa
It can be expressed in terms of **pressure head as 7 m** of water
69 kPa = 9.81 kN/m^{3}x 7 m or
69 kPa=133kN/m^{3}x 0.518 m (or 518 mm)

γ_{Hg}= 133 kN/ m^{3,} γ_{water}= 9.81 kN/ m^3

a 7 m-tall column of water with a cross-sectional area of 6.5 cm^2 weighs 4.5 kg.

CHANGE OF PRESSURE IN STATIC FLUIDS : INCOMPRESSIBLE FLUIDS

$$\boxed{\frac{dp}{dz} = -\gamma}$$

$$\gamma = \rho g$$

most engineering applications the variation in g is negligible

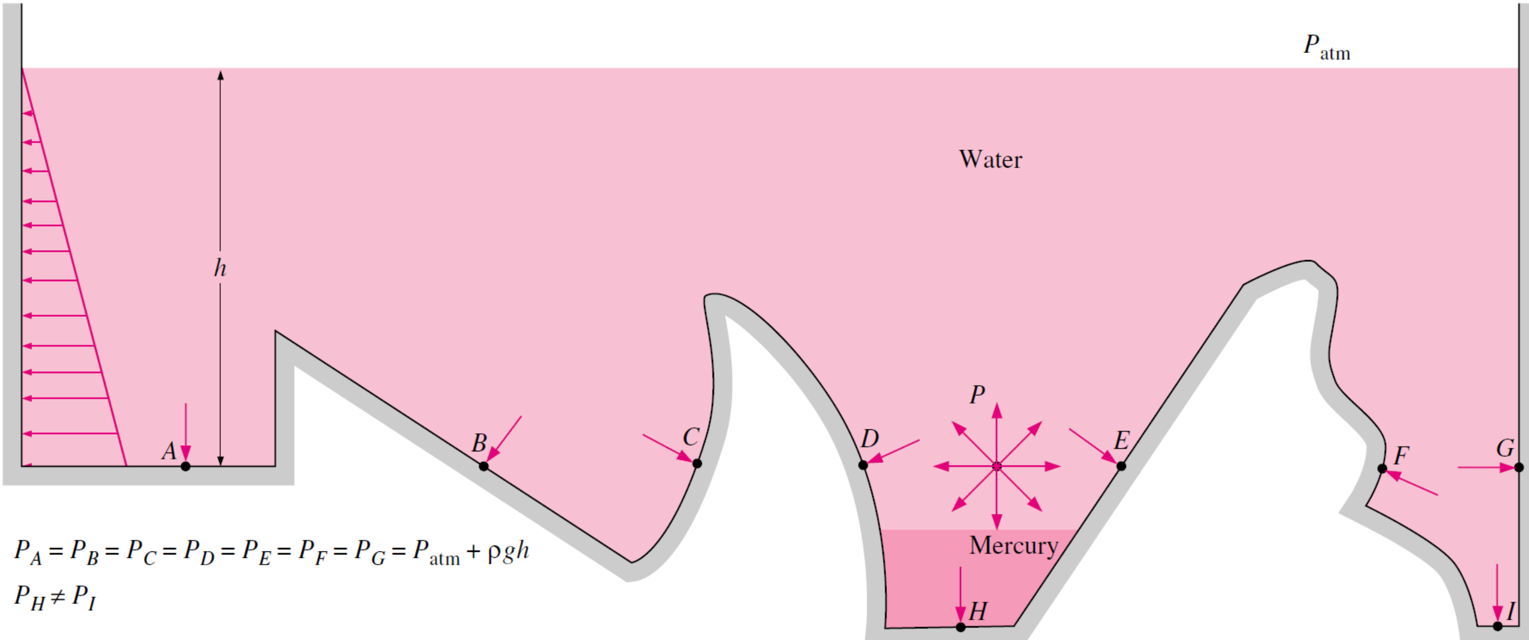
In general, a fluid with constant density is called an **incompressible fluid**.

$$\int_{p_1}^{p_2} dp = -\gamma \int_{z_1}^{z_2} dz$$

$$p_2 - p_1 = -\gamma(z_2 - z_1)$$

$$p_1 - p_2 = \gamma\underbrace{(z_2 - z_1)}_{h}$$

$$p_1 - p_2 = \gamma h$$



P_{atm}
Water
h
P
Mercury
A
B
C
D
E
F
G
H
I
$P_A = P_B = P_C = P_D = P_E = P_F = P_G = P_{atm} + \rho g h$
$P_H \neq P_I$

CHANGE OF PRESSURE: COMPRESSIBLE FLUIDS

Gases such as air, oxygen, and nitrogen as being **compressible fluids** because the density of the gas can change significantly with modest changes in pressure and temperature.

$$\boxed{\frac{dp}{dz} = -\gamma}$$

it is necessary to consider the possible variation in γ before the equation can be integrated

Note: Specific weight of the gases so small compared to liquids variation of pressure with elevation change is negligible.

$$p = \rho R T$$

Ideal Gas Law

$$\frac{dp}{dz} = -\frac{gp}{RT}$$

$$\int_{p_1}^{p_2} \frac{dp}{p} = \ln \frac{p_2}{p_1} = -\frac{g}{R} \int_{z_1}^{z_2} \frac{dz}{T} \Longrightarrow p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

g and R are assumed to be constant over the elevation change from z_1 to z_2.

Assumption: the temperature has a constant value T_0 over the range z_1 to z_2 (isothermal conditions),

CHANGE OF PRESSURE: COMPRESSIBLE FLUIDS

In 2010, the world's tallest building, the Burj Khalifa skyscraper was completed and opened in the United Arab Emirates. The final height of the building, which had remained a secret until completion, is 828 m.

Find: a) Estimate the ratio of the pressure at the 828 m top of the building to the pressure at its base, assuming the air to be at a common temperature of 290°K. b) Compare the pressure calculated in part (a) with that obtained by assuming the air to be incompressible with γ=11.99 N/m3 at 1 atm (abs) (values for air at standard sea level conditions).

a

$$\begin{aligned}\frac{p_2}{p_1} &= \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right] \\ &= \exp\left\{-\frac{(9.81\ \mathrm{m/s^2})(828\ \mathrm{m})}{(8.314\ \mathrm{J/mol\cdot\ K})(290\ ^\circ\mathrm{K})}\right\} \\ &= 0.0344\end{aligned}$$

b

$$p_2 = p_1 - \gamma(z_2 - z_1)$$

$$\begin{aligned}\frac{p_2}{p_1} &= 1 - \frac{\gamma(z_2 - z_1)}{p_1} \\ &= 1 - \frac{(11.99\ \mathrm{N/m^3})(828\ \mathrm{m})}{(101352\ \mathrm{N/m^2})} = 0.902\end{aligned}$$

Note that there is little difference between the two results. Since the pressure difference between the bottom and top of the building is small, it follows that the variation in fluid density is small and, therefore, the compressible fluid and incompressible fluid analyses yield essentially the same result.