ENVE 2061
Basic Fluid Mechanics

BUOYANCY, FLOTATION & STABILITY

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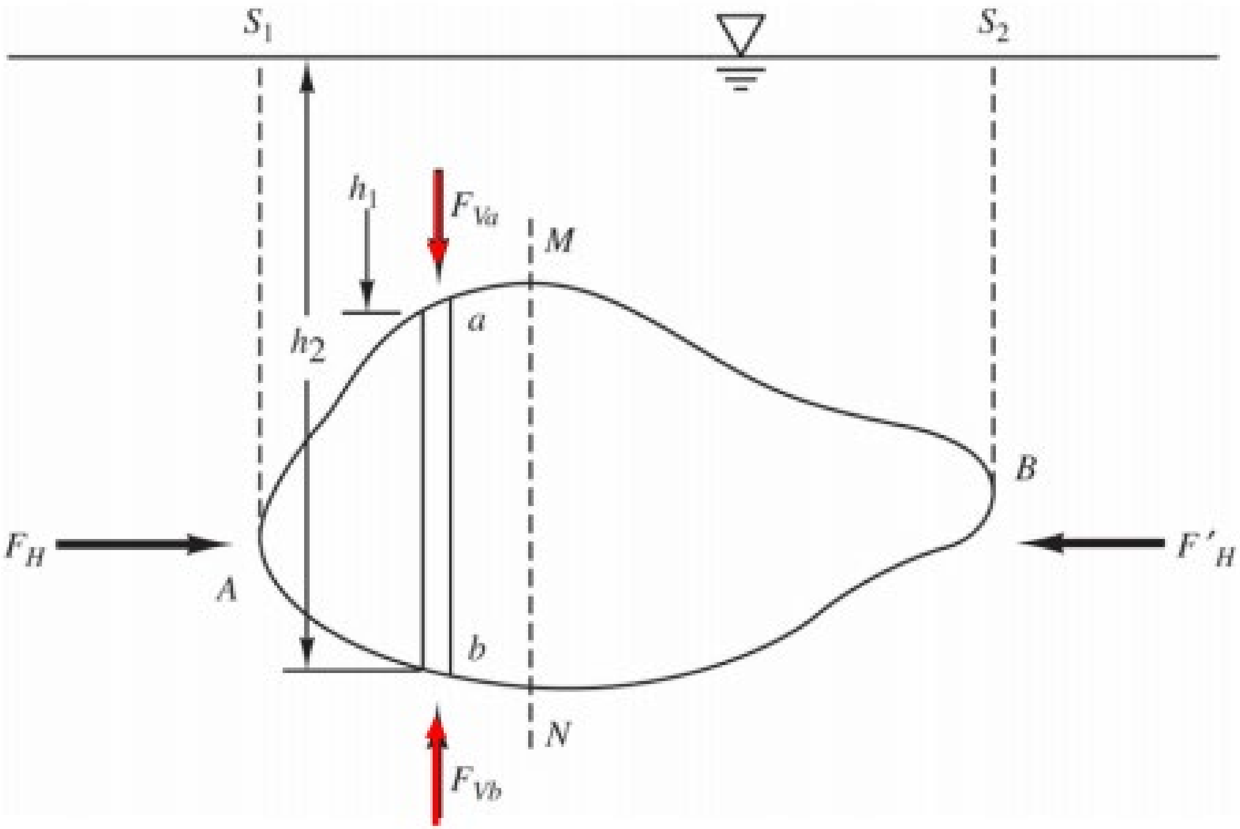
BUOYANCY FORCE

Archimedes discovered that ***the weight of a submerged body is reduced by an amount equal to the weight of the liquid displaced by the body***



BUOYANCY FORCE

Archimedes discovered that ***the weight of a submerged body is reduced by an amount equal to the weight of the liquid displaced by the body***

Vertical pressure force component:

Vertical pressure force on the top of the strip = $\gamma_{water} h_1 dA$ (downward)

Vertical pressure force on the bottom of the strip = $\gamma_{water} h_2 dA$ (upward)

Resultant vertical force component = $F_V = \gamma h_2 dA - \gamma h_1 dA$

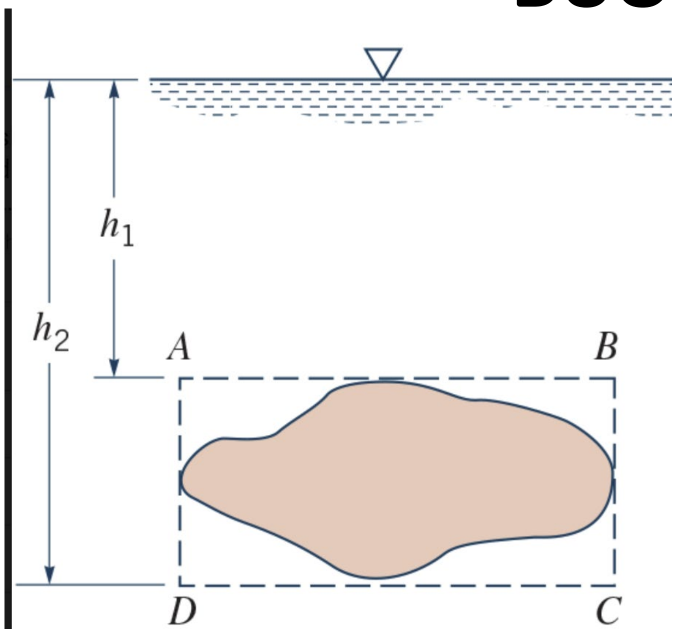
$$= \gamma (h_2 - h_1) dA$$

F_V = Weight of the water column "ab" replaced by the prism.

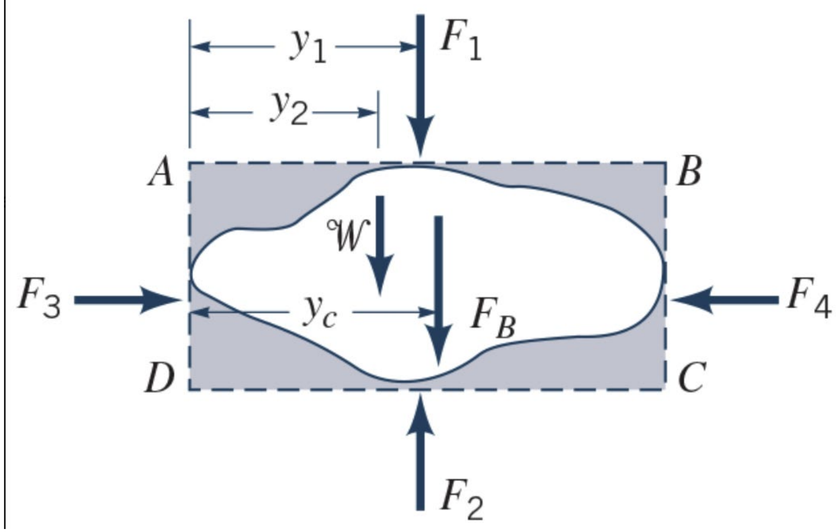
$$F_b = \gamma_{liquid} \forall_{displaced}$$

$$F_b = F_V$$

BUOYANCY FORCE



(a)



F_1, F_2, F_3, and F_4 are simply the forces exerted on the plane surfaces of the parallelepiped (for simplicity the forces in the x direction are not shown)

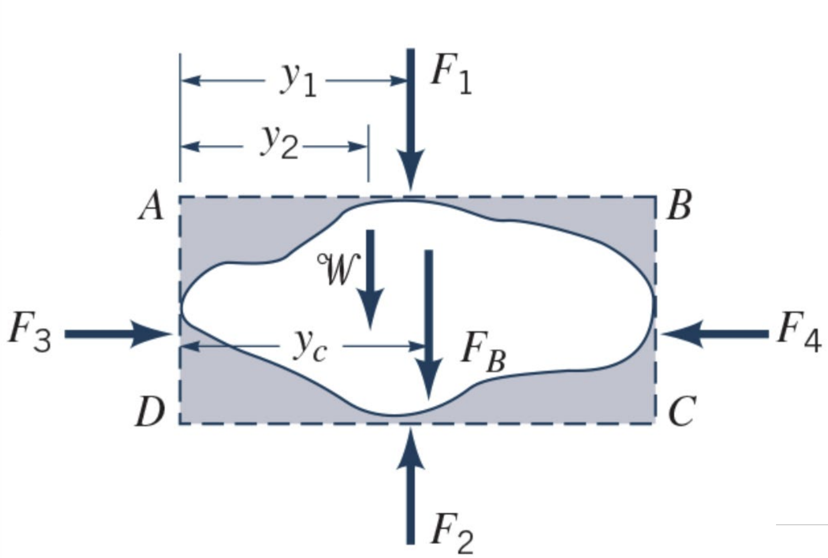
The forces on the vertical surfaces, such as F3 and F4, are all equal and cancel, so the equilibrium equation of interest is in the z direction

W is the weight of the shaded fluid volume (parallelepiped minus body)

$$F_B = F_2 - F_1 - \mathscr{W}$$

FB is the force the body is exerting on the fluid

BUOYANCY FORCE

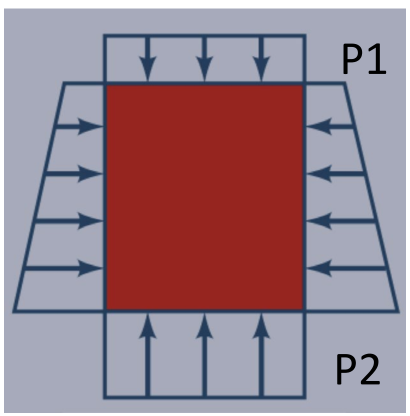


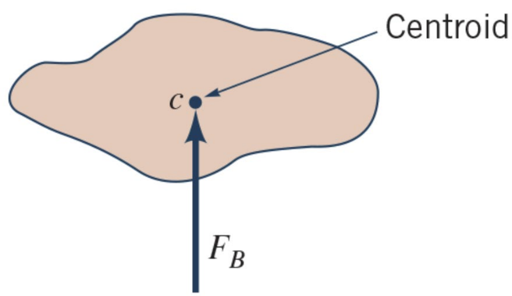
$$F_B = F_2 - F_1 - \mathscr{W}$$

$$F_2 - F_1 = \gamma(h_2 - h_1)A$$

$$F_B = \gamma(h_2 - h_1)A - \gamma[(h_2 - h_1)A - \mathcal{V}]$$

$$\boxed{F_B = \gamma \mathcal{V}}$$



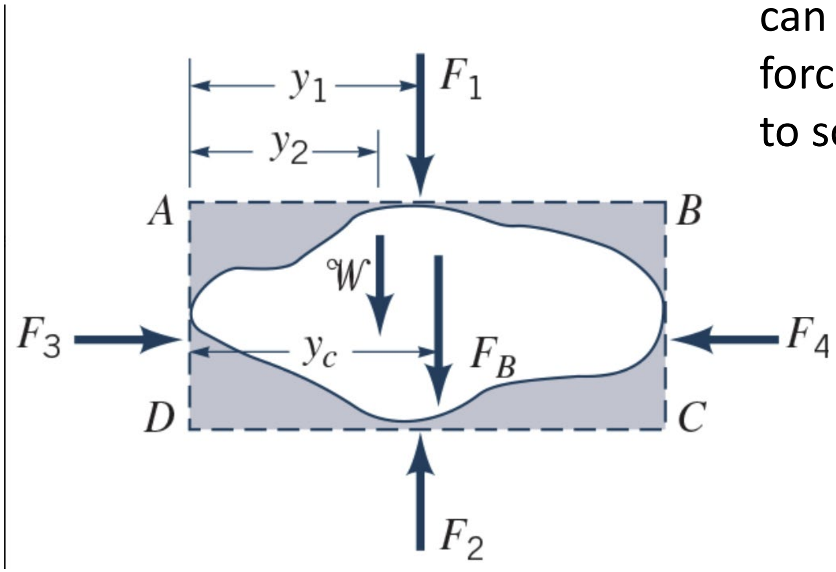


Therefore, the **buoyant force has a magnitude equal to the weight of the fluid displaced by the body and is directed vertically upward.**

This result is commonly referred to as *Archimedes' principle in honor of Archimedes (287–212 B.C.), a Greek mechanician and mathematician who first enunciated the basic ideas associated with hydrostatics.*

BUOYANCY FORCE

Buoyant Force Line of Action



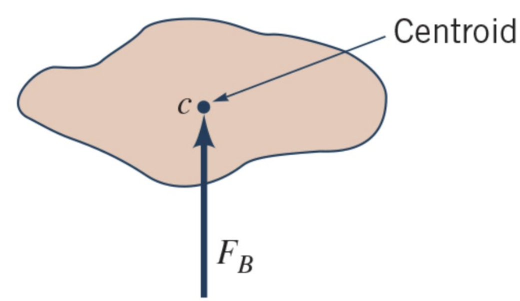
The location of the line of action of the buoyant force can be determined by summing moments of the forces shown on the free-body diagram with respect to some convenient axis

$$F_B y_c = F_2 y_1 - F_1 y_1 - \mathcal{W} y_2$$

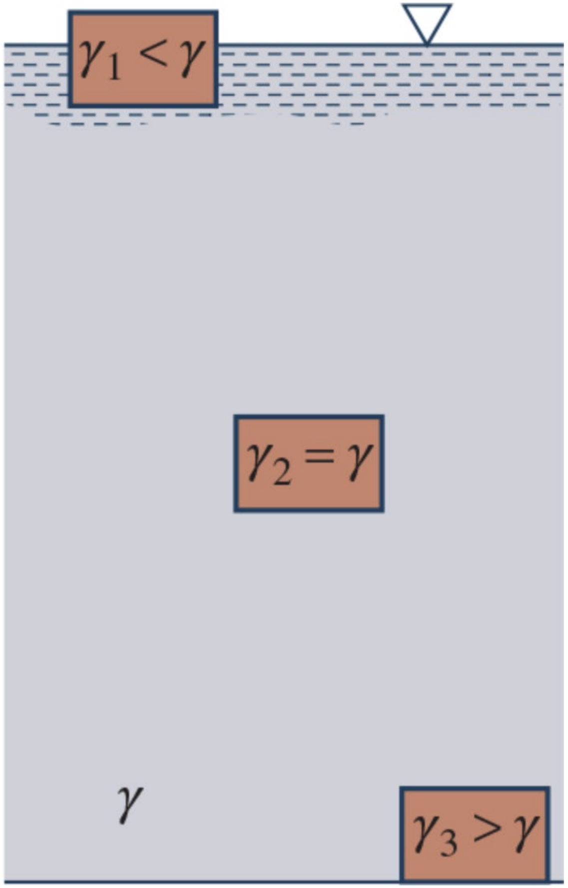
$$\mathcal{V} y_c = \mathcal{V}_T y_1 - (\mathcal{V}_T - \mathcal{V}) y_2$$

Buoyant force passes through the centroid of the displaced volume

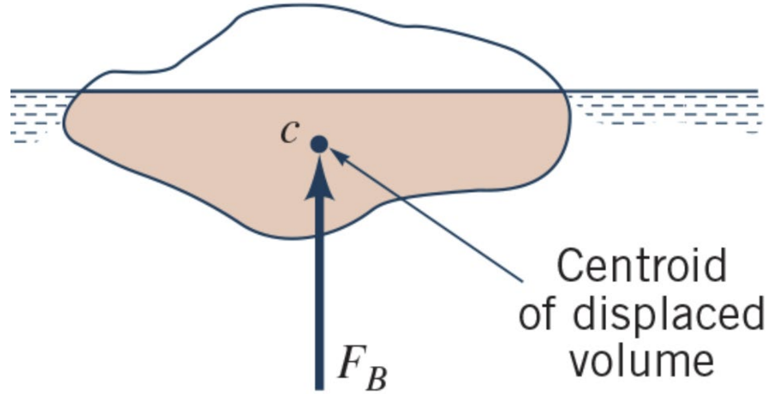
The point through which the buoyant force acts is called the center of buoyancy.



BUOYANCY FORCE



The effects of the specific weight (or density) of the body as compared to that of the surrounding fluid

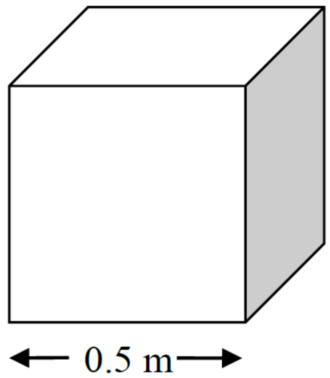


Floating bodies partially submerged

If the specific weight of the fluid above the liquid surface is very small compared with the liquid in which the body floats.

Since the fluid above the surface is usually air, for practical purposes this condition is satisfied.

Example16.1



A cube, 0.50 m on a side, is made of bronze having a specific weight of 86.9 kN/m^3. Determine the magnitude and direction of the force required to hold the cube in equilibrium completely.

a) in water

b) in mercury, $(s.g)_{mercury}$=13.54

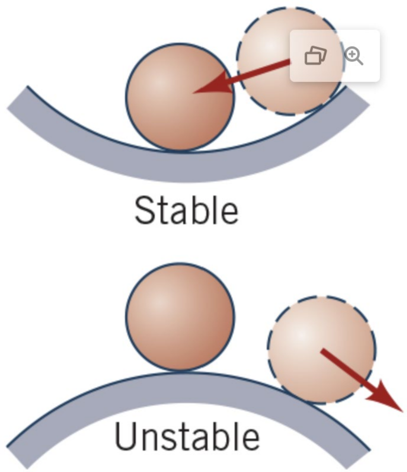
A piece of irregularly shaped metal weighs 301 N. When the metal is completely submerged in water, it weighs 253 N. Determine the specific weight and the specific gravity of the metal.

An iceberg (specific gravity 0.917) floats in the ocean (specific gravity 1.025). What percent of the volume of the iceberg is under water?

A floating 1m thick piece of ice sinks 0.025 m, with a 227 kg polar bear in the center of the ice. That is the area of the ice in the plane of the water level? For seawater, S =1.03.

A river barge, whose cross is approximately rectangular, carries a load of grain. The barge is 8.5 m wide and 27.5 m long. When unloaded, its draft (depth of submergence) is 1.5 m and with the load of grain the draft is 2.1 m. Determine **(a)** the unloaded weight of the barge and **(b)** the weight of the grain.

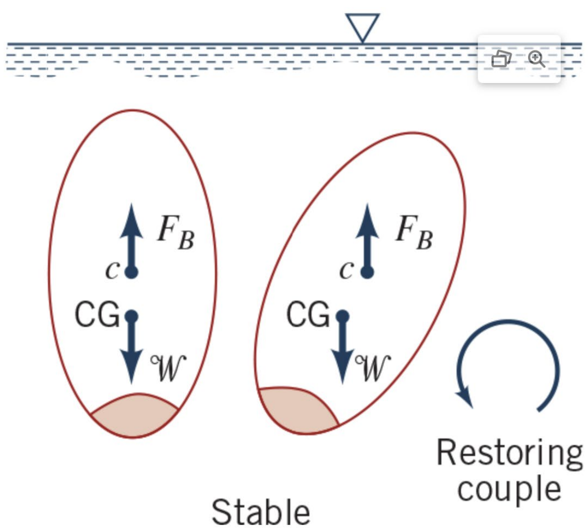
STABILITY



Stable: a body is said to be in a stable equilibrium position if, when displaced, it returns to its equilibrium position.

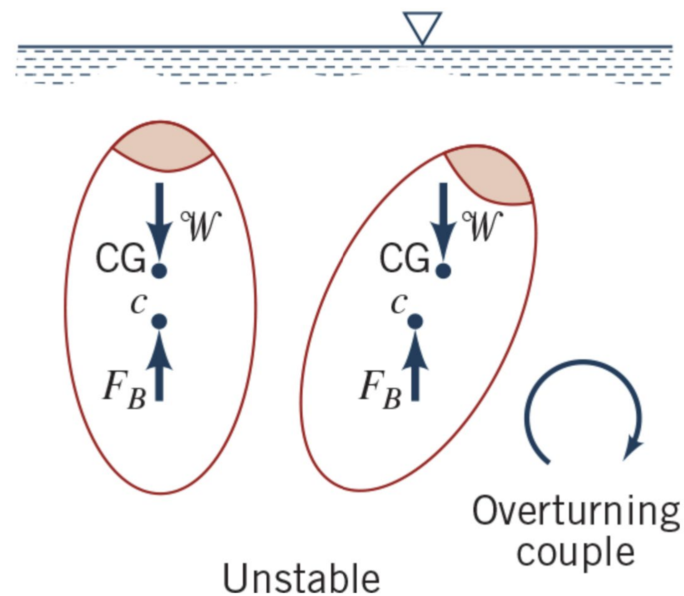
Unstable: it is in an unstable equilibrium position if, when displaced (even slightly), it moves to a new equilibrium position

Stability considerations are particularly important for submerged or floating bodies because **the centers of buoyancy** and **gravity** do not necessarily **coincide.**

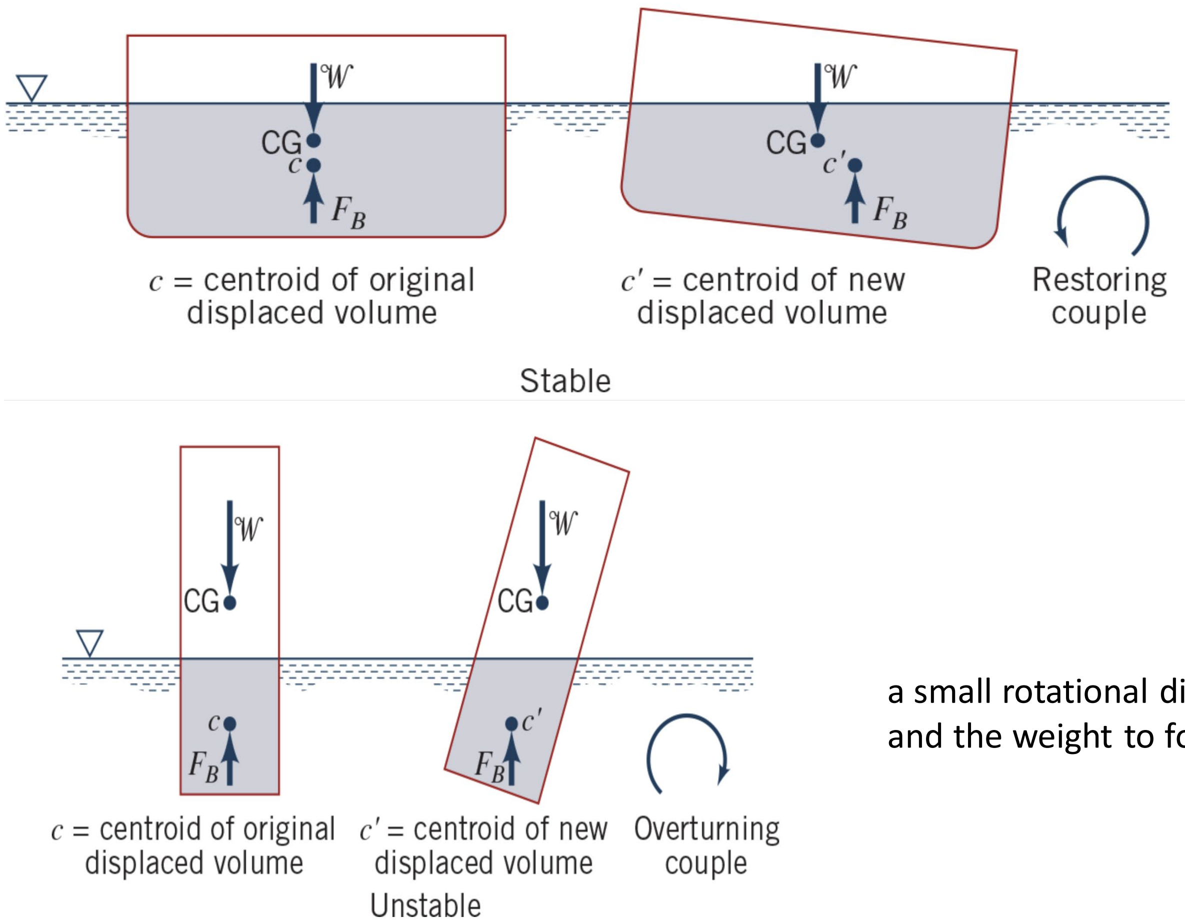


CG below centroid of F_B

CG above centroid of F_B



STABILITY

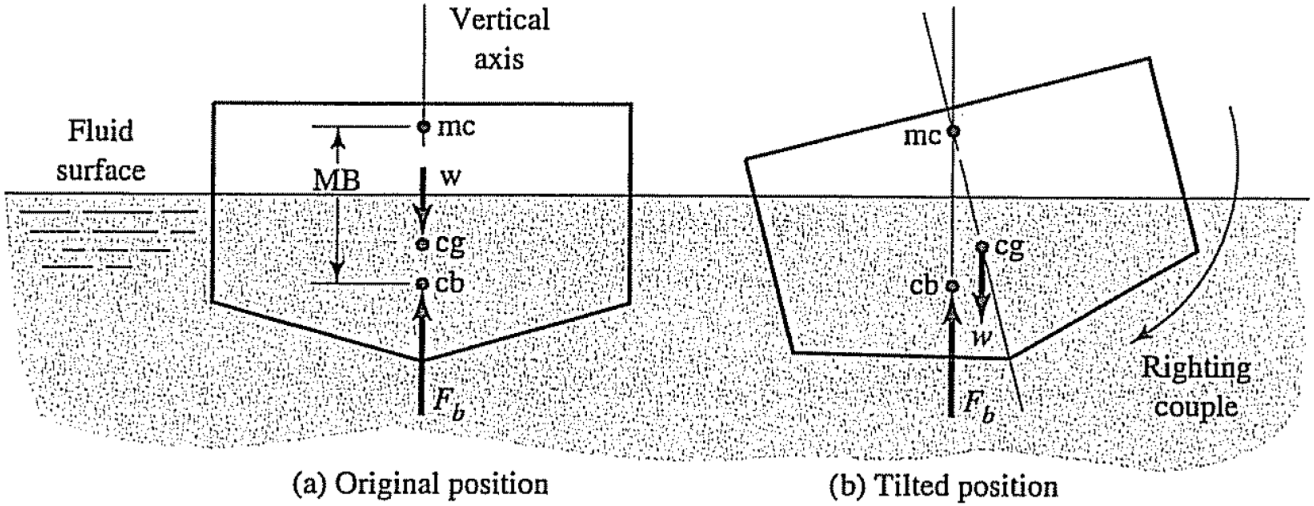


For floating bodies, the stability problem is more complicated because as the **body rotates the location of the center of buoyancy** (which is the centroid of the displaced volume) may change

As the body rotates the buoyant force, FB, shifts to pass through the centroid of the newly formed displaced volume and, combines with the weight to form a couple that will cause the body to return to its original equilibrium position.

a small rotational displacement can cause the buoyant force and the weight to form an overturning couple as illustrated.

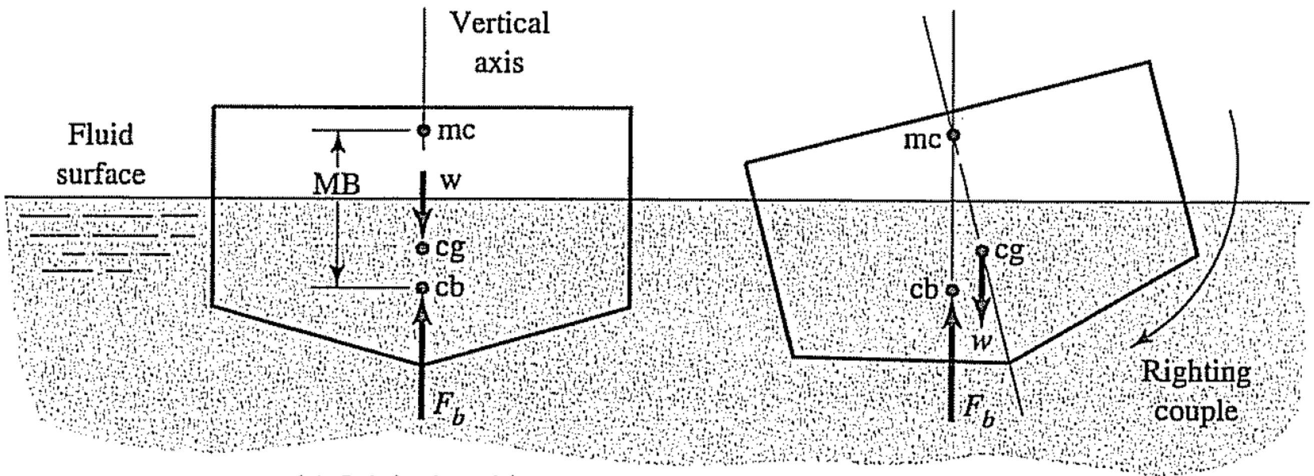
STABILITY



Metacenter (mc): Intersection of the vertical axis of a body when in its equilibrium position and a vertical line through the new position of the center of buoyancy when the body is rotated sligthly.

A floating body is stable if its center of gravity is below the metacenter.

STABILITY



(a) Original position (b) Tilted position

Stability of a floating body is determined by calculation of its metacenter;

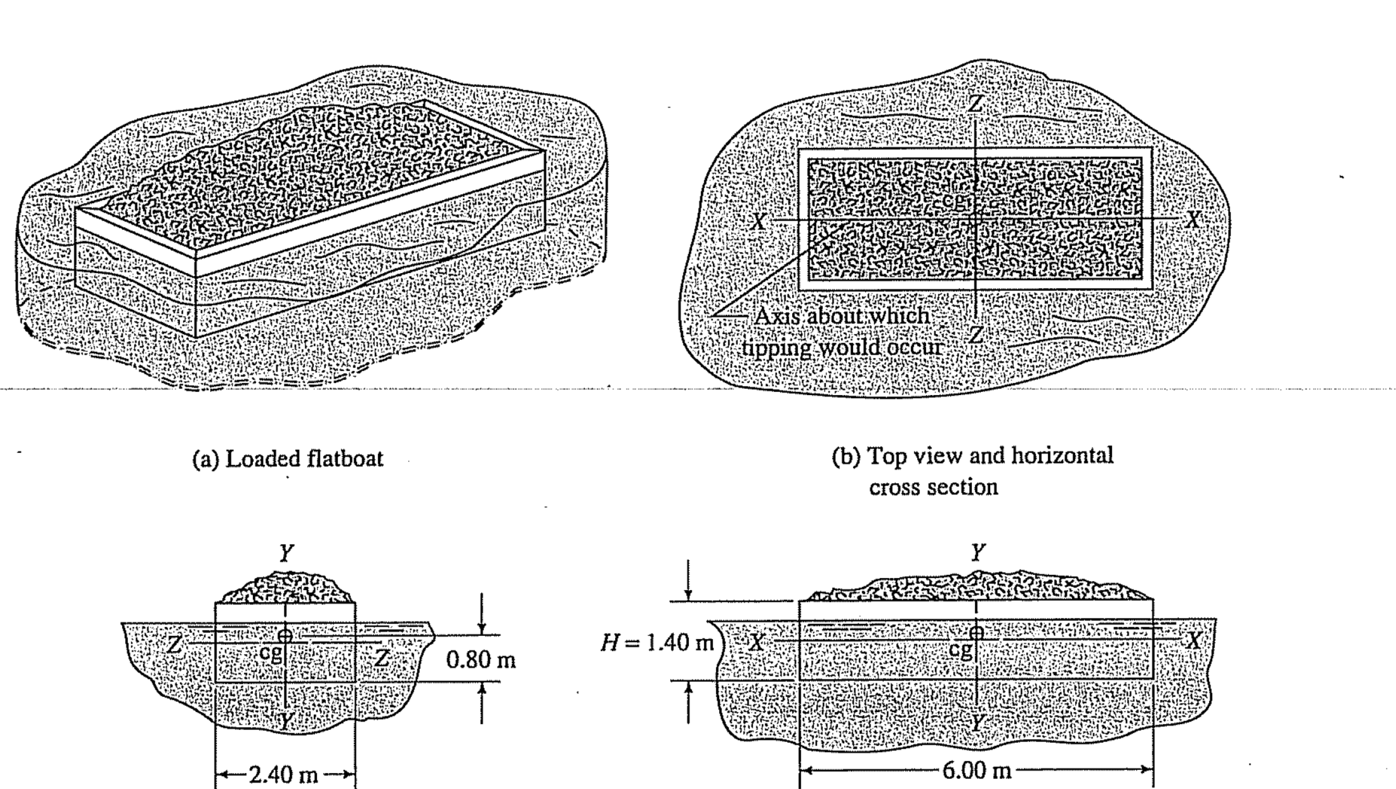
MB= I/Vd

I=Least moment of inertia of a horizontal section of the body taken at the surface of the liquid
Vd=Displaced volume of fluid

PROCEDURE FOR EVALUATING THE STABILITY OF FLOATING BODIES

1. Determine the position of the floating body, using the principles of buoyancy.
2. Locate the center of buoyancy, cb; compute the distance from some reference axis to cb, called y_{cb}. Usually, the bottom of the object is taken as the reference axis.
3. Locate the center of gravity, cg; compute y_{cg} measured from the same reference axis.
4. Determine the shape of the area at the fluid surface and compute the *smallest* moment of inertia I for that shape.
5. Compute the displaced volume V_d.
6. Compute $MB = I/V_d$.
7. Compute $y_{mc} = y_{cb} + MB$.
8. If $y_{mc} > y_{cg}$, the body is stable.
9. If $y_{mc} < y_{cg}$, the body is unstable.

Example Problem 5.5 (Applied Fluid Mechanics)
A floatboat hull that weighs 150 kN. Determine the boat is stable in water



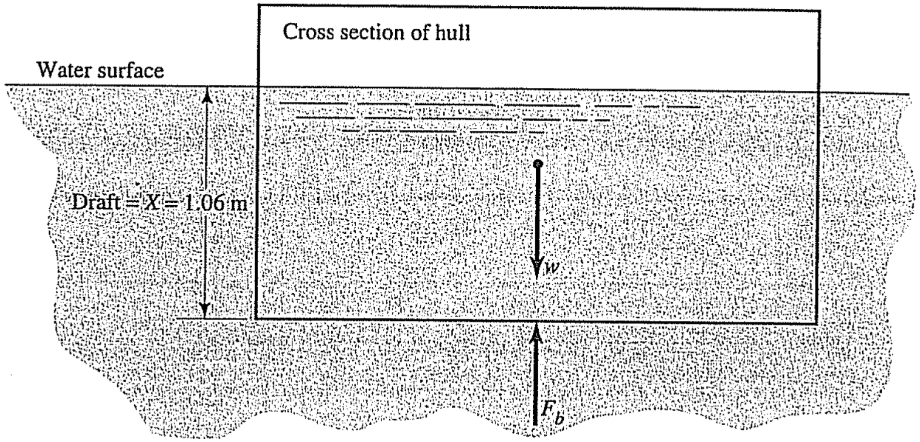
(a) Loaded flatboat

(b) Top view and horizontal cross section

(c) Front view and vertical cross section

(d) Side view

Free-body diagram.



it floats with 1.06 m submergence.

Equation of equilibrium: $$\sum F_v = 0 = F_B - W$$

Center of buoyancy ??

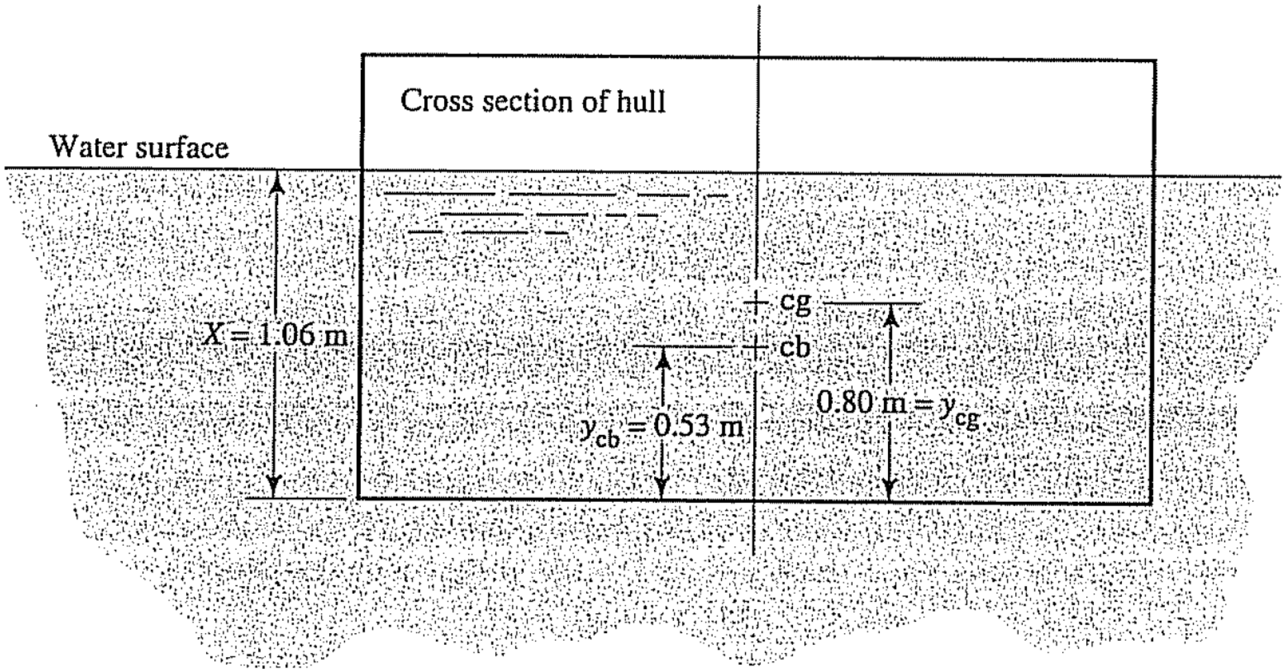
$$W = \dot{F}_B$$

Submerged Volume: $$V_d = B \times L \times x$$

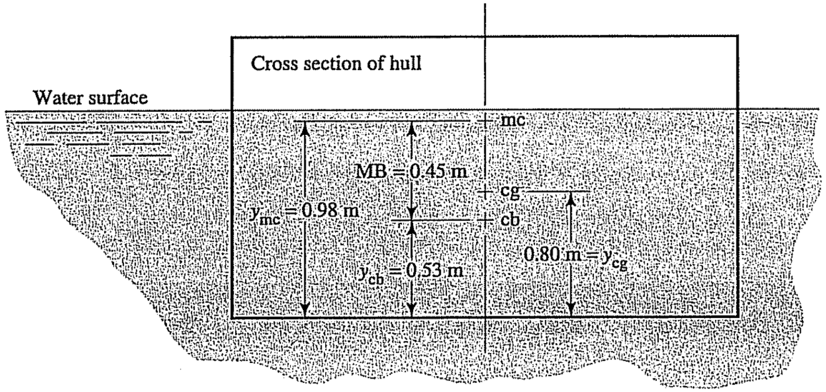
Buoyant Force: $$F_B = \gamma_f V_d = y_f \times B \times LxX$$

$$W = \hbar = \gamma_f \times B \times L \times X$$

$$x = \frac{w}{BxLx\gamma_f} = \frac{150\ kN}{(2.4\ m)(6.0\ m)} x \frac{m^3}{(9.81\ kN)} = 1.06\ m$$



It is at the center of the displaced volüme of water. In this case, it is on the vertical axis od the boat at a distamce of 0.53 m from the bottom. That is the half of the draft, X. Ycb=0.53 m.



Because the center of gravity is above the center of buoyancy, metacenter should be located to determine whether the boat is stable.

$$\mathrm{M}_B = I/V_d$$

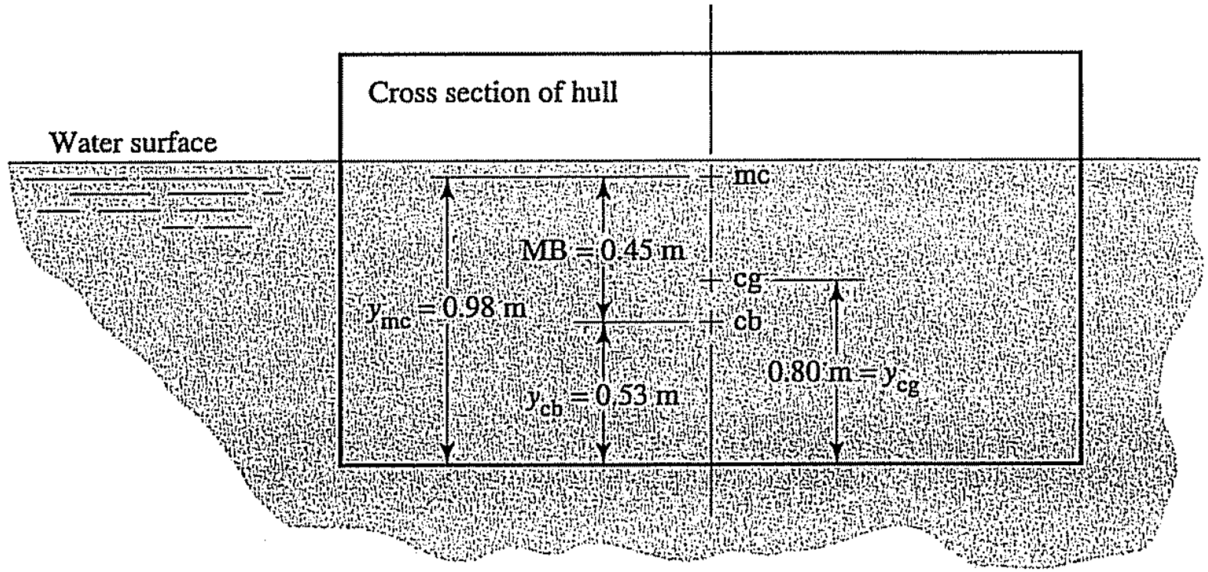
$$V_d = L \times B \times X = (6.0m)(2.4m)(1.06m) = 15.26m^3$$

The moment of inertia I is determined about the axis X-X because this would yield the smallest value for I:

$$I = \frac{LB^3}{12} = \frac{(6.0m)(2.4m)^3}{12} = 6.91m^4$$

$$MB = I/V_d = 6.91\,m^4/15.26\,m^3 = 0.45m$$

$$y_{m_C} = MB + y_C = 0.53 + 0.45 \text{ m} = 0.98 \text{ m}$$



$y_{mc} > y_{cg}$ ➔ Because the metacenter is above the center of gravity, the boat is stable