ENVE2061
Basic Fluid Mechanics

Elif Soyer

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Introduction - Some Characteristics of Fluids

Dimensions & dimensional homogeneity

System of Units

Introduction - Some Characteristics of Fluids

- What is a fluid?
- What is the difference between a solid and a fluid?

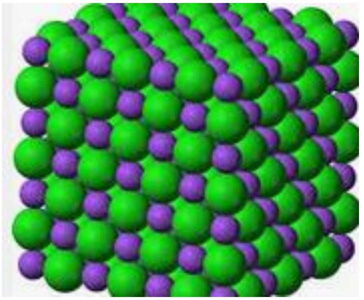
A **solid** is "hard" and not easily deformed, whereas a **fluid** is "soft" and is easily deformed

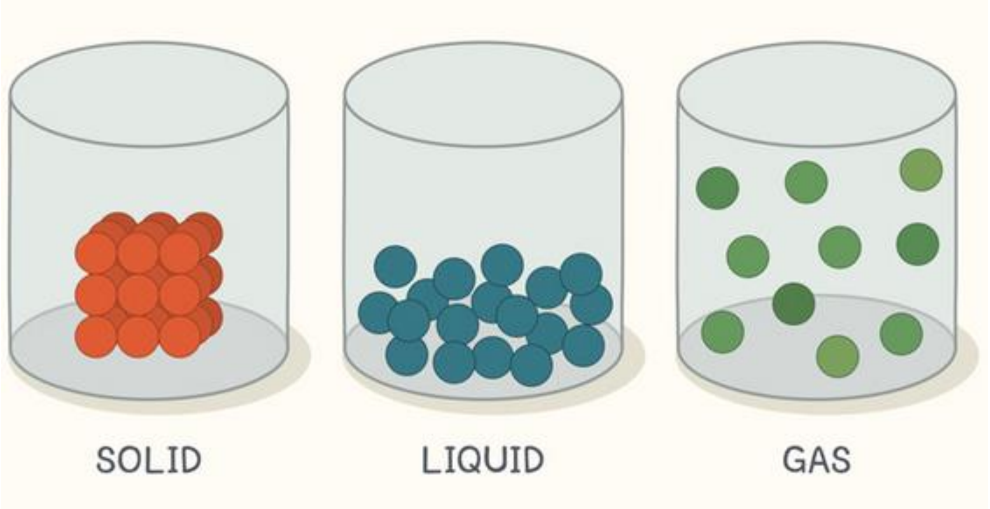
Both liquids and gases are FLUIDS

Example: we can readily move through air.

This definition is quite descriptive, but not very satisfactory from a scientific or engineering point of view.

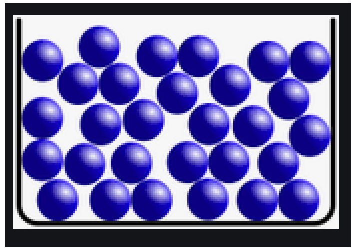
A closer look at the molecular structure of materials
SOLIDS

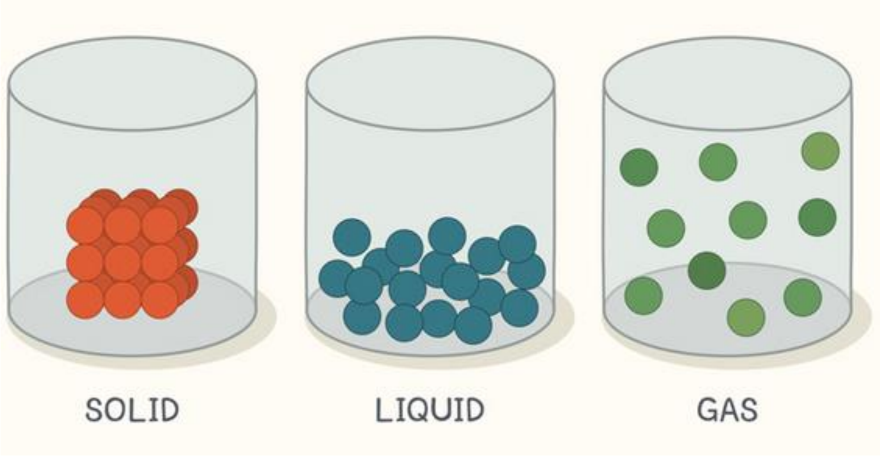




A **solid** such as steel, concrete, etc.
has densely spaced molecules with large intermolecular cohesive forces that allow the solid to maintain its shape, and to not be easily deformed.

A closer look at the molecular structure of materials
LIQUIDS





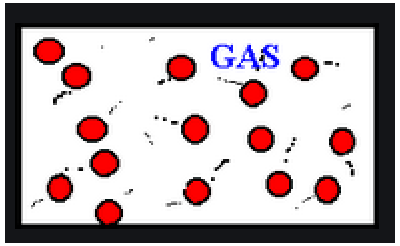
In liquids such as water, oil, etc. the molecules are spaced farther apart, the intermolecular forces are smaller than for solids, and the molecules have more freedom of movement.

Thus, **liquids can be easily deformed but not easily compressed**.

They can be poured into containers.

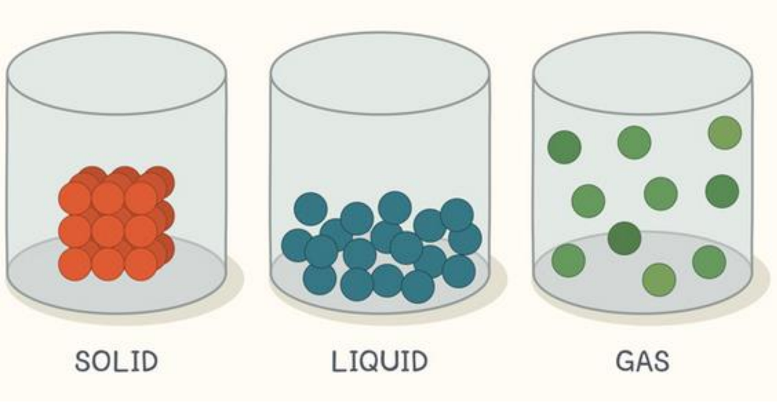
They can be forced through a tube.

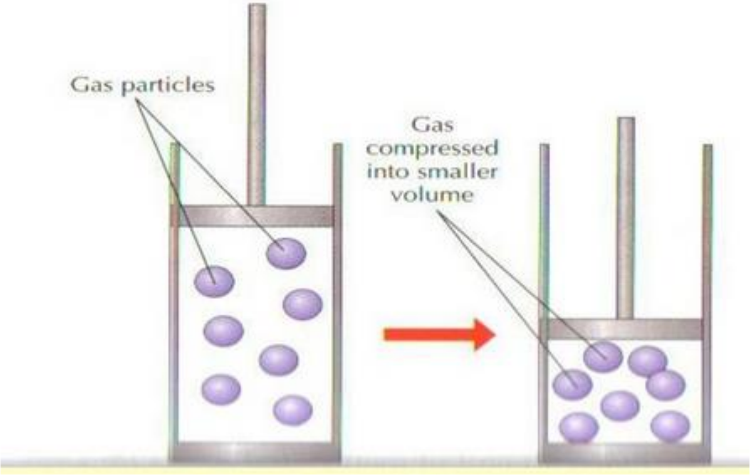
A closer look at the molecular structure of materials
GASES

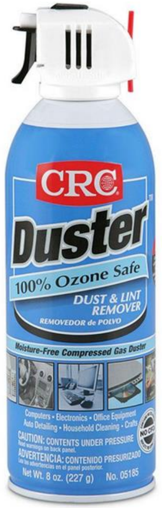


Gases such as air, oxygen, etc. have even greater molecular spacing and freedom of motion with negligible cohesive intermolecular forces.

As a result, they are easily deformed and compressed and will completely fill the volume of any container in which they are placed.



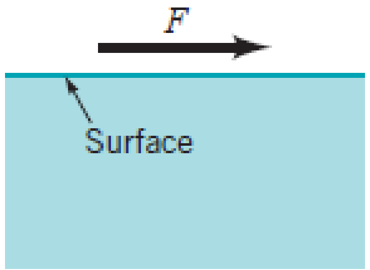




Both <u>liquids</u> and <u>gases</u> are FLUIDS.

Differences between solids and fluids

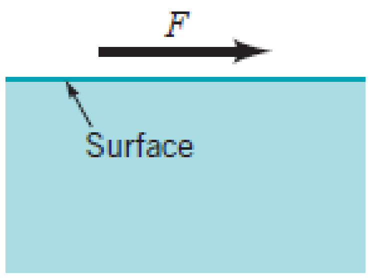
Although the differences between solids and fluids can be explained qualitatively on the basis of molecular structure, a more specific distinction is based on **how they deform under the action of an external load**.



A **fluid** is defined as a substance that deforms continuously when acted on by a shearing stress of any magnitude.

Differences between solids and fluids

A shearing stress, force per unit area, is created whenever a tangential force acts on a surface as shown by the figure.



Common **solids** such as steel or other metals will **initially deform** (usually very small deformation), **but they will not continuously deform** or in other words **they will not FLOW**.

However, common fluids such as water, oil, and air satisfy the definition of a fluid—that is, they will flow when acted on by a shearing stress.

Other materials
Rheology

Some materials, such as slurries, tar, putty, toothpaste, are not easily classified since they will behave as a solid if the applied shearing stress is small, but if the stress exceeds some critical value, the substance will flow.

The study of such materials is called *rheology*.

Dimensions & dimensional homogeneity

Fluid mechanics we will be dealing with a variety of fluid characteristics.
Thus, it is necessary to develop a system for describing these characteristics both ***qualitatively*** and ***quantitatively***.

The **qualitative aspect** serves to identify the nature, or type of the characteristics (such as length, time, stress, and velocity).

Whereas the **quantitative aspect** provides a numerical measure of the characteristics.

Dimensions & dimensional homogeneity

The quantitative description requires both a number and a standard by which various quantities can be compared.

- A standard for length might be a meter or foot,
- for time an hour or second,
- for mass a slug or kilogram.

Such standards are called **units,** and several systems of units are in common use.

Dimensions & dimensional homogeneity

The qualitative description is given in terms of certain **primary quantities**, such as length, L, time, T, mass, M, and temperature, Θ.

These primary quantities can then be used to provide a qualitative description of any other **secondary quantity**:

for example,

$$\text{area} \doteq L^2 \qquad \text{velocity} \doteq LT^{-1} \qquad \text{density} \doteq ML^{-3}$$

the symbol $\doteq$

is used to indicate the *dimensions* of the secondary quantity in terms of the primary quantities.

Dimensions & dimensional homogeneity

to describe qualitatively a velocity, V, we would write

$$V \doteq LT^{-1}$$

and say that "the dimensions of a velocity equal length divided by time."

Dimensions & dimensional homogeneity

The primary quantities are also referred to as ***basic dimensions.***

For a wide variety of problems involving fluid mechanics, only the three basic dimensions, L, T, and M are required. Alternatively, L, T, and F could be used, where F is the basic dimensions of force. Since Newton's law states that force is equal to mass times acceleration, it follows that $F \doteq MLT^{-2}$ or $M \doteq FL^{-1}T^2$. Thus, secondary quantities expressed in terms of M can be expressed in terms of F through the relationship above. For example, stress, σ, is a force per unit area, so that $\sigma \doteq FL^{-2}$, but an equivalent dimensional equation is $\sigma \doteq ML^{-1}T^{-2}$.

3 basic dimensions : L, T, M or L, T, F

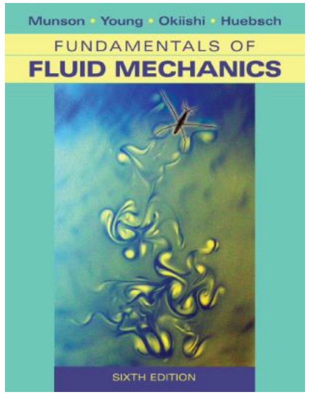
Newton's Law : Force = mass * Acceleration

$$F = M \frac{L}{T^2} \Rightarrow M = F \frac{T^2}{L}$$

TABLE 1.1

Dimensions Associated with Common Physical Quantities

	FLT System	*MLT* System
Acceleration	LT^{-2}	LT^{-2}
Angle	$F^0L^0T^0$	$M^0L^0T^0$
Angular acceleration	T^{-2}	T^{-2}
Angular velocity	T^{-1}	T^{-1}
Area	L^2	L^2
Density	$FL^{-4}T^2$	ML^{-3}
Energy	FL	ML^2T^{-2}
Force	F	MLT^{-2}
Frequency	T^{-1}	T^{-1}
Heat	FL	ML^2T^{-2}
Length	L	L
Mass	$FL^{-1}T^2$	M
Modulus of elasticity	FL^{-2}	$ML^{-1}T^{-2}$
Moment of a force	FL	ML^2T^{-2}
Moment of inertia (area)	L^4	L^4
Moment of inertia (mass)	FLT^2	ML^2
Momentum	FT	MLT^{-1}
Power	FLT^{-1}	ML^2T^{-3}
Pressure	FL^{-2}	$ML^{-1}T^{-2}$
Specific heat	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	FL^{-3}	$ML^{-2}T^{-2}$
Strain	$F^0L^0T^0$	$M^0L^0T^0$
Stress	FL^{-2}	$ML^{-1}T^{-2}$
Surface tension	FL^{-1}	MT^{-2}
Temperature	Θ	Θ
Time	T	T
Torque	FL	ML^2T^{-2}
Velocity	LT^{-1}	LT^{-1}
Viscosity (dynamic)	$FL^{-2}T$	$ML^{-1}T^{-1}$
Viscosity (kinematic)	L^2T^{-1}	L^2T^{-1}
Volume	L^3	L^3
Work	FL	ML^2T^{-2}



Dimensions & dimensional homogeneity

All theoretically derived equations are ***dimensionally homogeneous***—that is, the dimensions of the left side of the equation must be the same as those on the right side, and all additive separate terms must have the same dimensions. We accept as a fundamental premise that all equations describing physical phenomena must be dimensionally homogeneous. If this were not true, we would be attempting to equate or add unlike physical quantities, which would not make sense. For example, the equation for the velocity, V, of a uniformly accelerated body is

$$V = V_0 + at$$

where V_0 is the initial velocity, a the acceleration, and t the time interval. In terms of dimensions the equation is

$$LT^{-1} \doteq LT^{-1} + LT^{-1}$$

and thus Eq. is dimensionally homogeneous.

Dimensions & dimensional homogeneity

Some equations that are known to be valid contain constants having dimensions. The equation for the distance, d, traveled by a freely falling body can be written as

$$d = 16.1t^2$$

and a check of the dimensions reveals that the constant must have the dimensions of LT^{-2} if the equation is to be dimensionally homogeneous. Actually, Eq. is a special form of the well-known equation from physics for freely falling bodies,

$$d = \frac{gt^2}{2}$$

in which g is the acceleration of gravity.

Dimensions & dimensional homogeneity

$$\boxed{d = 16.1t^2}$$

$$\boxed{d = \frac{gt^2}{2}}$$

Equation $d = \frac{gt^2}{2}$ is dimensionally homogeneous and valid in any system of units.

For $g = 32.2 \text{ ft/s}^2$ the equation reduces to $d = 16.1t^2$ and is valid only for the system of units using feet and seconds.

Equations that are restricted to a particular system of units can be denoted as *restricted homogeneous equations*,

equations valid in any system of units, are *general homogeneous equations*.

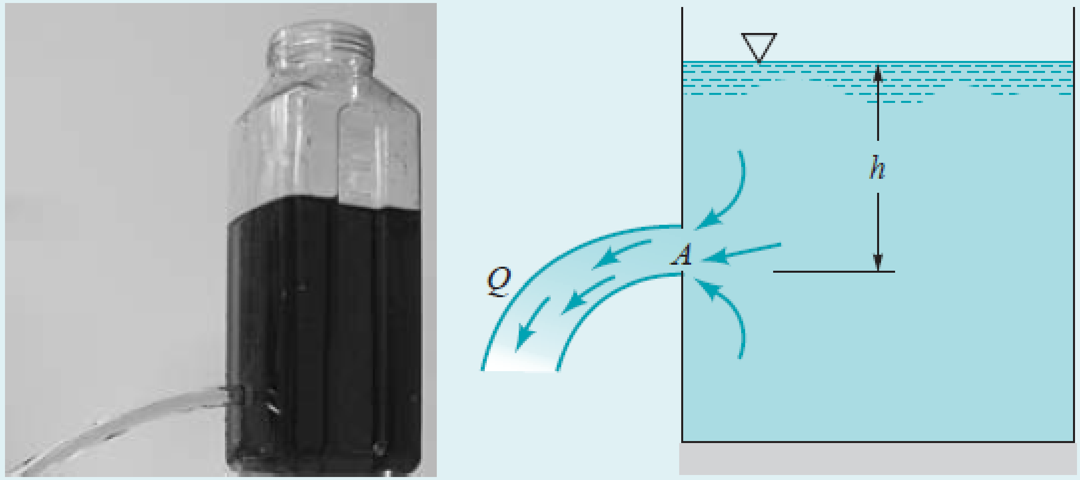
EXAMPLE 1.1 Restricted and General Homogeneous Equations

GIVEN A liquid flows through an orifice located in the side of a tank as shown in Fig. E1.1. A commonly used equation for determining the volume rate of flow, Q, through the orifice is

$$Q = 0.61\,A\sqrt{2gh}$$

where A is the area of the orifice, g is the acceleration of gravity, and h is the height of the liquid above the orifice.

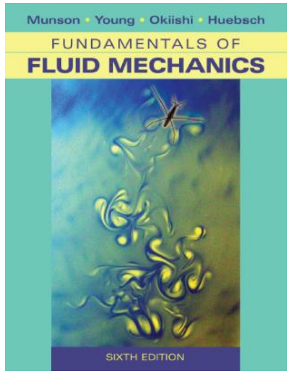
FIND Investigate the dimensional homogeneity of this formula.



(a) (b)

■ FIGURE E1.1

SOLUTION

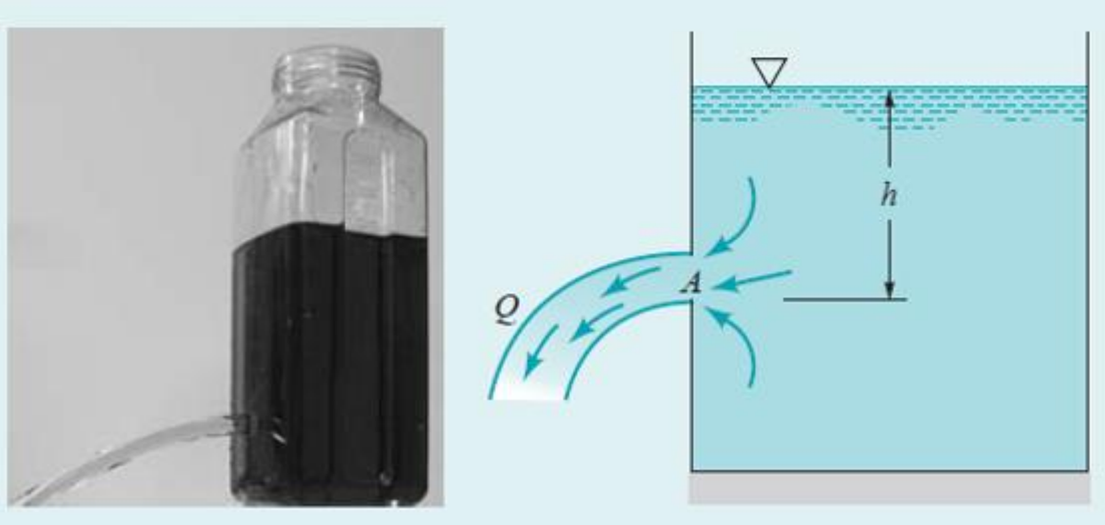


$$Q = 0.61\,A\sqrt{2gh}$$

the volume rate of flow, Q, through the orifice is

$$Q = 0.61\,A\sqrt{2gh}$$

where A is the area of the orifice, g is the acceleration of gravity, and h is the height of the liquid above the orifice.



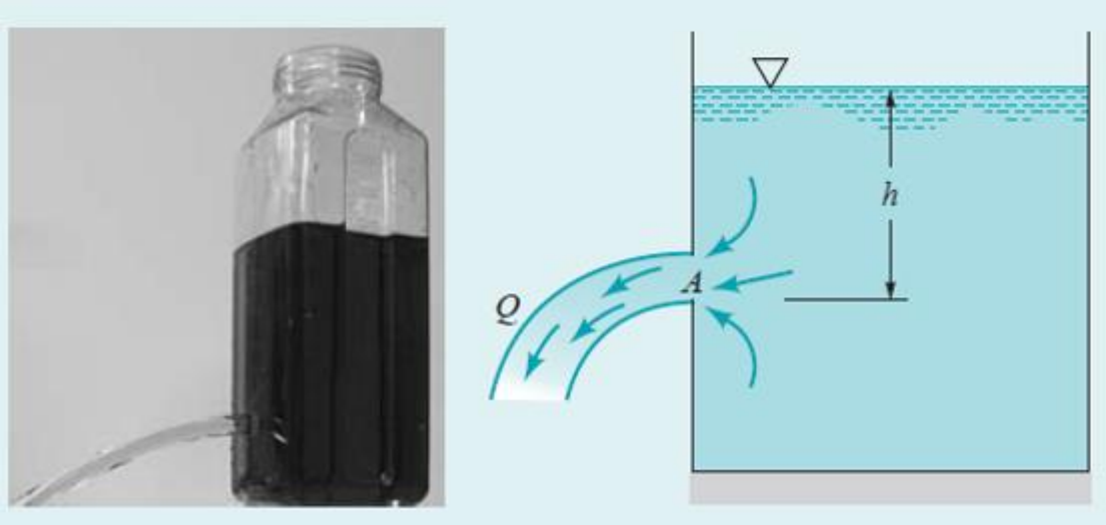
Q: volume rate of flow $\frac{L^3}{T}$

A: area L^2

g: acceleration of gravity $\frac{L}{T^2}$

h: height L

$$Q = 0.61\,A\sqrt{2gh}$$



$$Q = 0.61\,A\sqrt{2gh} \Longrightarrow \frac{L^3}{T} = 0.61\,\sqrt{2}\;L^2\left[\frac{L}{T^2}\,L\right]^{1/2}$$

the equation is dimensionally homogeneous $\Longleftarrow$ both sides of the formula have the same dimensions $L^3\,T^{-1}$

System of Units

In addition to the qualitative description of the various quantities of interest, it is generally necessary to have a quantitative measure of any given quantity.

For example, if we measure the width of a page in a book and say that it is 10 units wide, the statement has no meaning until the unit of length is defined.

If we indicate that the unit of length is a meter, and define the meter as some standard length, a unit system for length has been established and a numerical value can be given to the page width.

In addition to length, a unit must be established for each of the remaining basic quantities force, mass, time, and temperature.

System of Units

There are several systems of units in use and we will consider three systems that are commonly used in engineering.

- International System (SI)
- British Gravitational System (BG)
- English Engineering System (EE)

System of Units – International System (SI)

International System (SI). In 1960 the Eleventh General Conference on Weights and Measures, the international organization responsible for maintaining precise uniform standards of measurements, formally adopted the *International System of Units* as the international standard. This system, commonly termed SI, has been widely adopted worldwide and is widely used (although certainly not exclusively) in the United States. It is expected that the long-term trend will be for all countries to accept SI as the accepted standard and it is imperative that engineering students become familiar with this system. In SI the unit of length is the meter (m), the time unit is the second (s), the mass unit is the kilogram (kg), and the temperature unit is the kelvin (K). Note that there is no degree symbol used when expressing a temperature in kelvin units. The kelvin temperature scale is an absolute scale and is related to the Celsius (centigrade) scale (°C) through the relationship

$$K = °C + 273.15$$

Although the Celsius scale is not in itself part of SI, it is common practice to specify temperatures in degrees Celsius when using SI units.

System of Units – International System (SI)

The force unit, called the newton (N), is defined from Newton's second law as

$$1 \text{ N} = (1 \text{ kg})(1 \text{ m/s}^2)$$

Thus, a 1-N force acting on a 1-kg mass will give the mass an acceleration of 1 m/s^2. Standard gravity in SI is 9.807 m/s^2 (commonly approximated as 9.81 m/s^2) so that a 1-kg mass weighs 9.81 N under standard gravity. Note that weight and mass are different, both qualitatively and quantitatively! The unit of *work* in SI is the joule (J), which is the work done when the point of application of a 1-N force is displaced through a 1-m distance in the direction of a force. Thus,

$$1 \text{ J} = 1 \text{ N} \cdot \text{m}$$

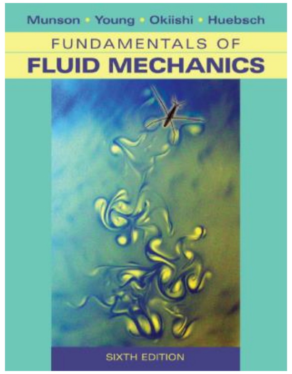
The unit of *power* is the watt (W) defined as a joule per second. Thus,

$$1 \text{ W} = 1 \text{ J/s} = 1 \text{ N} \cdot \text{m/s}$$

System of Units – International System (SI)

TABLE 1.2
Prefixes for SI Units

Factor by Which Unit Is Multiplied	Prefix	Symbol	Factor by Which Unit Is Multiplied	Prefix	Symbol
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	k	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10	deka	da	10^{-18}	atto	a
10^{-1}	deci	d			



System of Units – British Gravitational (BG) System

British Gravitational (BG) System. In the BG system the unit of length is the foot (ft), the time unit is the second (s), the force unit is the pound (lb), and the temperature unit is the degree Fahrenheit (°F) or the absolute temperature unit is the degree Rankine (°R), where

$$°\mathrm{R} = °\mathrm{F} + 459.67$$

The mass unit, called the *slug*, is defined from Newton's second law (force = mass × acceleration) as

$$1 \text{ lb} = (1 \text{ slug})(1 \text{ ft/s}^2)$$

This relationship indicates that a 1-lb force acting on a mass of 1 slug will give the mass an acceleration of 1 ft/s^2.

The weight, $\mathcal{W}$ (which is the force due to gravity, g) of a mass, m, is given by the equation

$$\mathcal{W} = mg$$

and in BG units

$$\mathcal{W}(\text{lb}) = m(\text{slugs})\,g(\text{ft/s}^2)$$

Since the earth's standard gravity is taken as $g = 32.174$ ft/s^2 (commonly approximated as 32.2 ft/s^2), it follows that a mass of 1 slug weighs 32.2 lb under standard gravity.

System of Units – English Engineering (EE) System

English Engineering (EE) System. In the EE system, units for force *and* mass are defined independently; thus special care must be exercised when using this system in conjunction with Newton's second law. The basic unit of mass is the pound mass (lbm), and the unit of force is the pound (lb).[1] The unit of length is the foot (ft), the unit of time is the second (s), and the absolute temperature scale is the degree Rankine (°R). To make the equation expressing Newton's second law dimensionally homogeneous we write it as

$$\mathbf{F} = \frac{m\mathbf{a}}{g_c}$$

where g_c is a constant of proportionality which allows us to define units for both force and mass. For the BG system, only the force unit was prescribed and the mass unit defined in a consistent manner such that $g_c = 1$. Similarly, for SI the mass unit was prescribed and the force unit defined in a consistent manner such that $g_c = 1$. For the EE system, a 1-lb force is defined as that force which gives a 1 lbm a standard acceleration of gravity which is taken as 32.174 ft/s^2. Thus, for Eq. to be both numerically and dimensionally correct

$$1 \text{ lb} = \frac{(1 \text{ lbm})(32.174 \text{ ft/s}^2)}{g_c}$$

[1] It is also common practice to use the notation, lbf, to indicate pound force.

System of Units – English Engineering (EE) System

so that

$$g_c = \frac{(1\ \text{lbm})(32.174\ \text{ft/s}^2)}{(1\ \text{lb})}$$

With the EE system, weight and mass are related through the equation

With the EE system, weight and mass are related through the equation

$$\mathscr{W} = \frac{mg}{g_c}$$

where g is the local acceleration of gravity. Under conditions of standard gravity ($g = g_c$) the weight in pounds and the mass in pound mass are numerically equal. Also, since a 1-lb force gives a mass of 1 lbm an acceleration of 32.174 ft/s^2 and a mass of 1 slug an acceleration of 1 ft/s^2, it follows that

$$1\ \text{slug} = 32.174\ \text{lbm}$$

System of Units

	Standard International (SI)	British Gravitational (BG)
Length	Meter (m)	Foot (ft)
Time	Second (s)	Second (s)
Mass	Kilogram (kg)	Slug
Temperature	Kelvin (K) $K = {}^{o}C + 273.15$	Rankine (^{o}R) ${}^{o}R = {}^{o}F + 459.67$
Force	Newton (N)	Pound (lb)
Work (Energy)	Joule (J) 1 J = 1 N.m	lb.ft BTU (British Thermal Unit)
Power	Watt (W) 1 W = 1 J/s = 1 N.m/s	lb.ft/s Hp (Horsepower)

TABLE 1.4

Conversion Factors from SI Units to BG and EE Units[a]

	To Convert from	to	Multiply by
Acceleration	m/s^2	ft/s^2	3.281
Area	m^2	ft^2	1.076 E + 1
Density	kg/m^3	lbm/ft^3	6.243 E − 2
	kg/m^3	$slugs/ft^3$	1.940 E − 3
Energy	J	Btu	9.478 E − 4
	J	ft · lb	7.376 E − 1
Force	N	lb	2.248 E − 1
Length	m	ft	3.281
	m	in.	3.937 E + 1
	m	mile	6.214 E − 4
Mass	kg	lbm	2.205
	kg	slug	6.852 E − 2
Power	W	ft · lb/s	7.376 E − 1
	W	hp	1.341 E − 3
Pressure	N/m^2	in. Hg (60 °F)	2.961 E − 4
	N/m^2	lb/ft^2 (psf)	2.089 E − 2
	N/m^2	$lb/in.^2$ (psi)	1.450 E − 4
Specific weight	N/m^3	lb/ft^3	6.366 E − 3
Temperature	°C	°F	$T_F = 1.8\,T_C + 32°$
	K	°R	1.800
Velocity	m/s	ft/s	3.281
	m/s	mi/hr (mph)	2.237
Viscosity (dynamic)	$N \cdot s/m^2$	$lb \cdot s/ft^2$	2.089 E − 2
Viscosity (kinematic)	m^2/s	ft^2/s	1.076 E + 1
Volume flowrate	m^3/s	ft^3/s	3.531 E + 1
	m^3/s	gal/min (gpm)	1.585 E + 4

[a]If more than four-place accuracy is desired, refer to Appendix E.

TABLE 1.3

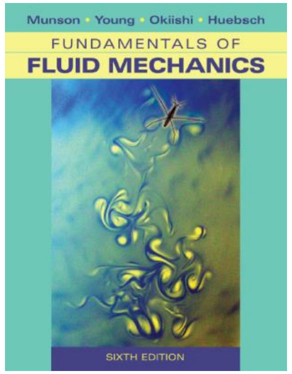
Conversion Factors from BG and EE Units to SI Units[a]

	To Convert from	to	Multiply by
Acceleration	ft/s^2	m/s^2	3.048 E − 1
Area	ft^2	m^2	9.290 E − 2
Density	lbm/ft^3	kg/m^3	1.602 E + 1
	$slugs/ft^3$	kg/m^3	5.154 E + 2
Energy	Btu	J	1.055 E + 3
	ft · lb	J	1.356
Force	lb	N	4.448
Length	ft	m	3.048 E − 1
	in.	m	2.540 E − 2
	mile	m	1.609 E + 3
Mass	lbm	kg	4.536 E − 1
	slug	kg	1.459 E + 1
Power	ft · lb/s	W	1.356
	hp	W	7.457 E + 2
Pressure	in. Hg (60 °F)	N/m^2	3.377 E + 3
	lb/ft^2 (psf)	N/m^2	4.788 E + 1
	$lb/in.^2$ (psi)	N/m^2	6.895 E + 3
Specific weight	lb/ft^3	N/m^3	1.571 E + 2
Temperature	°F	°C	$T_C = (5/9)(T_F - 32°)$
	°R	K	5.556 E − 1
Velocity	ft/s	m/s	3.048 E − 1
	mi/hr (mph)	m/s	4.470 E − 1
Viscosity (dynamic)	$lb \cdot s/ft^2$	$N \cdot s/m^2$	4.788 E + 1
Viscosity (kinematic)	ft^2/s	m^2/s	9.290 E − 2
Volume flowrate	ft^3/s	m^3/s	2.832 E − 2
	gal/min (gpm)	m^3/s	6.309 E − 5

[a]If more than four-place accuracy is desired, refer to Appendix E.

1.8 If V is a velocity, determine the dimensions of Z, α, and G, which appear in the dimensionally homogeneous equation

$$V = Z(\alpha - 1) + G$$



1.8 If V is a velocity, determine the dimensions of Z, α, and G, which appear in the dimensionally homogeneous equation

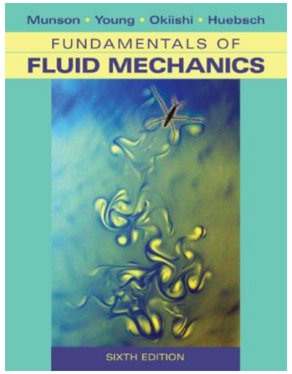
$$V = Z(\alpha - 1) + G$$

$$\frac{L}{T} \qquad \frac{L}{T} \qquad \frac{L}{T}$$

1.9 The volume rate of flow, Q, through a pipe containing a slowly moving liquid is given by the equation

$$Q = \frac{\pi R^4 \Delta p}{8 \mu \ell}$$

where R is the pipe radius, Δp the pressure drop along the pipe, μ a fluid property called viscosity $(FL^{-2}T)$, and ℓ the length of pipe. What are the dimensions of the constant $\pi/8$? Would you classify this equation as a general homogeneous equation? Explain.



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Q: volume rate of flow : $\frac{L^3}{T}$

R: pipe radius : L

Δp: pressure drop $\Rightarrow P_1 - P_2 \Rightarrow \Delta P$ should be in pressure unit.

Pressure: $\frac{\text{Force}}{\text{Area}}$: $\frac{F}{L^2}$

μ: viscosity : $\frac{F}{L^2 T}$

ℓ: length of pipe : L

1.9 The volume rate of flow, Q, through a pipe containing a slowly moving liquid is given by the equation

$$Q = \frac{\pi R^4 \Delta p}{8\mu\ell}$$

where R is the pipe radius, Δp the pressure drop along the pipe, μ a fluid property called viscosity $(FL^{-2}T)$, and ℓ the length of pipe. What are the dimensions of the constant $\pi/8$? Would you classify this equation as a general homogeneous equation? Explain.

$$\boxed{\frac{L^3}{T}} = \frac{\pi \; L^4 \; \frac{F}{L^2}}{8 \; \frac{FT}{L^2} \; L} = \frac{\pi}{8} \, L^{\cancel{4}\,3} \, \frac{\cancel{F}}{\cancel{L^2}} \, \frac{\cancel{L^2}}{\cancel{F}\,T} \cdot \frac{1}{\cancel{L}}$$

dimensionless

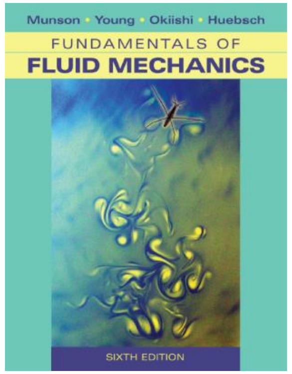
$$\frac{L^3}{T}$$

Equation is a general homogeneous equation ✓

1.10 According to information found in an old hydraulics book, the energy loss per unit weight of fluid flowing through a nozzle connected to a hose can be estimated by the formula

$$h = (0.04 \text{ to } 0.09)(D/d)^4 V^2/2g$$

where h is the energy loss per unit weight, D the hose diameter, d the nozzle tip diameter, V the fluid velocity in the hose, and g the acceleration of gravity. Do you think this equation is valid in any system of units? Explain.



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$$\boxed{h = (0.04 \text{ to } 0.09)\left(\frac{D}{d}\right)^4 \frac{V^2}{2g}}$$

h: energy loss per unit weight

D: hose diameter

d: nozzle tip diameter

V: fluid velocity in the hose

g: acceleration of gravity

energy loss $\Rightarrow E_1 - E_2 \Rightarrow$ energy units (dimensions)

↳ FL (Force * length)

weight : F (Force)

h: energy loss per unit weight $= \frac{FL}{F} = L$

D : hose diameter L : length

d : nozzle tip diameter L : length

D/d = dimensionless (L/L)

V: fluid velocity $\frac{L}{T}$ (length/time)

g : acceleration $\frac{L}{T^2}$ (length/time^2)

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$$h = \underbrace{(0.04 \text{ to } 0.09)}_{\text{constant}} \left(\frac{D}{d}\right)^4 \frac{V^2}{2g}$$

$$L = \frac{\text{constant}}{2} \left(\frac{L}{L}\right)^4 \frac{\left(\frac{L}{T}\right)^2}{\frac{L}{T^2}}$$

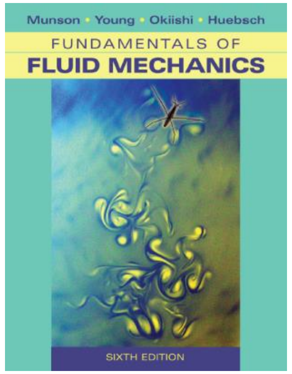
constant $\Rightarrow$ has no dimension

The equation is valid in any system of units (SI or BG can be used)

1.11 The pressure difference, Δp, across a partial blockage in an artery (called a *stenosis*) is approximated by the equation

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1\right)^2 \rho V^2$$

where V is the blood velocity, μ the blood viscosity $(FL^{-2}T)$, ρ the blood density (ML^{-3}), D the artery diameter, A_0 the area of the unobstructed artery, and A_1 the area of the stenosis. Determine the dimensions of the constants K_v and K_u. Would this equation be valid in any system of units?



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V: blood velocity

μ: blood viscosity $\left(\frac{FT}{L^2}\right)$

ρ: blood density $\left(\frac{M}{L^3}\right)$

D: artery diameter

A_0: area of the unobstructed artery

A_1: area of the stenosis

Dimensions of the constants K_v & K_u?

Would this eqn be valid in any system of units?

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V : velocity $\left(\frac{L}{T}\right)$

μ : viscosity $\left(\frac{FT}{L^2}\right)$

ρ : density $\left(\frac{M}{L^3}\right)$

D : diameter (L)

$\frac{A_0}{A_1}$: $\frac{\text{area}}{\text{area}}$: $\frac{L^2}{L^2}$ dimensionless

Δp : pressure difference : pressure unit (dimension)

: $\frac{\text{Force}}{\text{area}}$: $\frac{F}{L^2}$

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$$\frac{F}{L^2} = K_v \underbrace{\frac{\frac{F\cancel{T}}{L^2} \cdot \frac{L}{\cancel{T}}}{L}} + K_u \left(\text{dimensionless number} - 1\right)^2 \underbrace{\frac{M}{L^3}\left(\frac{L}{T}\right)^2}$$

$$\frac{F}{L^2}$$

$$\frac{M\cancel{L^2}}{\cancel{L^3}T^2} = \frac{M}{LT^2}$$

MLT → FLT
conversion required
(Newton's 2nd Law)

$$F = ma$$

$$F = M\frac{L}{T^2} \Rightarrow M = \frac{FT^2}{L}$$

$$\frac{M}{LT^2} = \frac{\frac{FT^2}{L}}{LT^2} = \frac{F}{L^2}$$

K_v & K_u ⇒ dimensionless,
equation is valid in any system of units

1.13 A formula to estimate the volume rate of flow, Q, flowing over a dam of length, B, is given by the equation

$$Q = 3.09\,BH^{3/2}$$

where H is the depth of the water above the top of the dam (called the head). This formula gives Q in $\mathrm{ft^3/s}$ when B and H are in feet. Is the constant, 3.09, dimensionless? Would this equation be valid if units other than feet and seconds were used?