ENVE2061
Basic Fluid Mechanics

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FLUID STATICS

Pressure at a Point

Basic Equation for Pressure Field

Pressure Variation in a Fluid at Rest

- Compressible Fluid
- Incompressible Fluid

Pressure Head

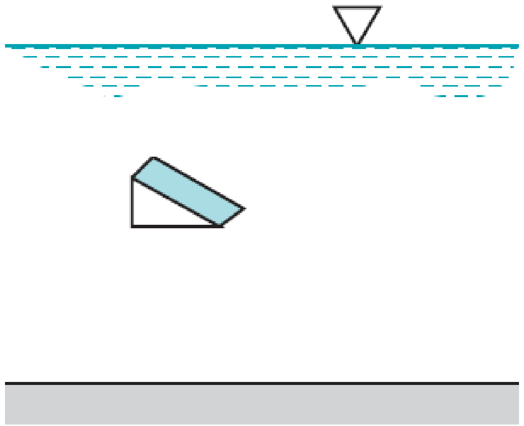
Absolute & Gage Pressure

FLUID STATICS

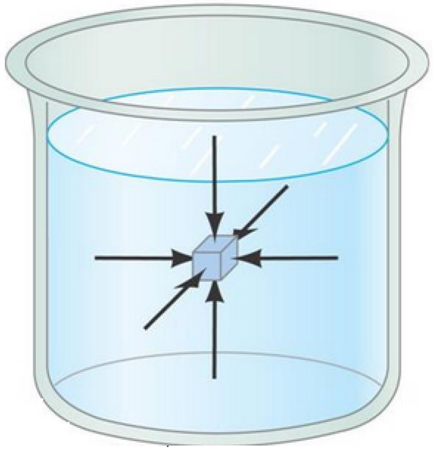
- determine the pressure at various locations in a fluid at rest.
- explain the concept of manometers and apply appropriate equations to determine pressures.
- calculate the hydrostatic pressure force on a plane or curved submerged surface.
- calculate the buoyant force and discuss the stability of floating or submerged objects.

Pressure at a Point

How the pressure at a point varies with the orientation of the plane passing through the point.

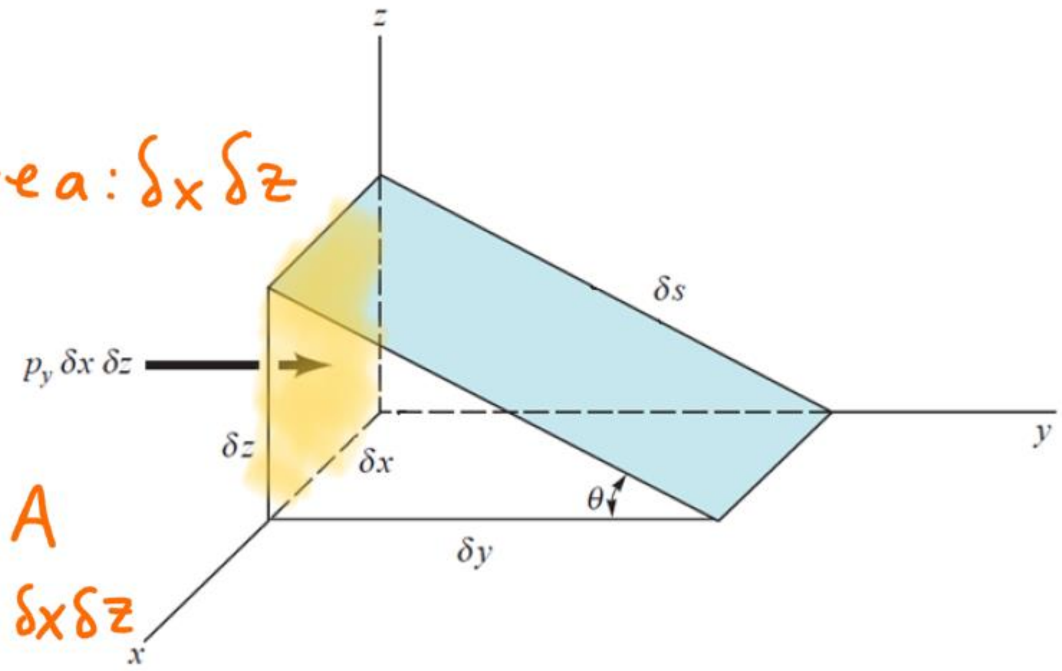


To answer this question, we consider the free-body diagram, that was obtained by removing a small triangular wedge of fluid from some arbitrary location within a fluid mass.



Pressure at a Point

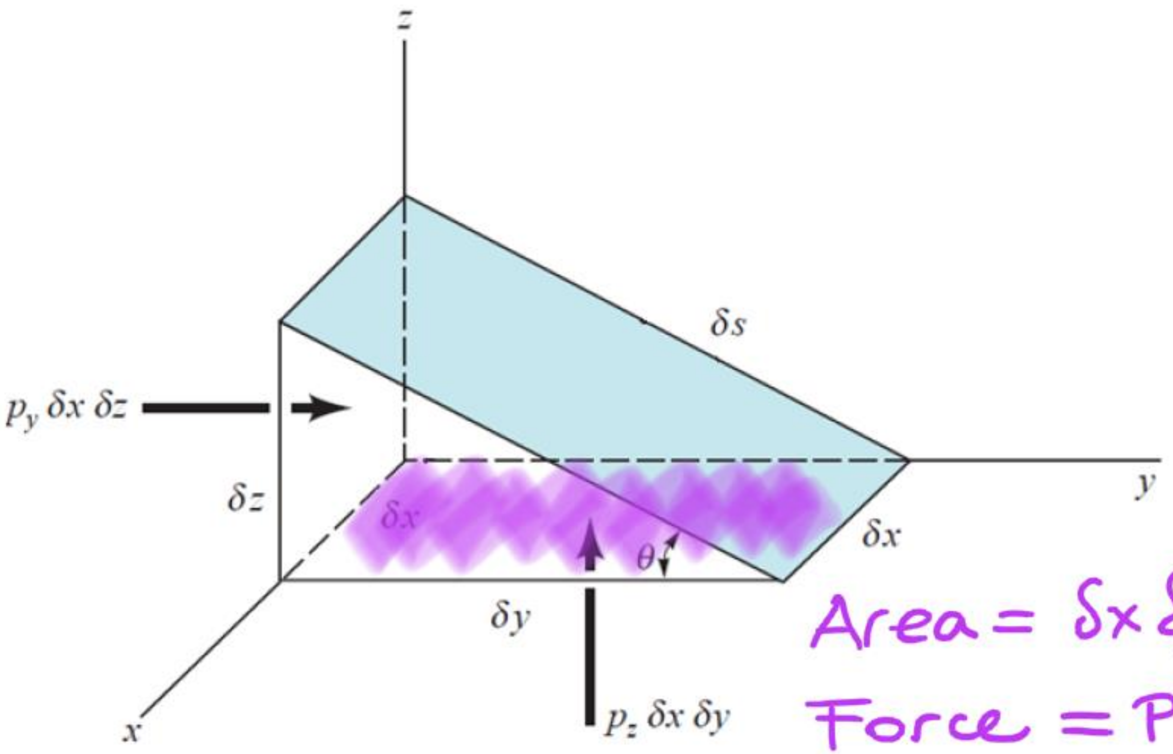
Area: $\delta x \, \delta z$



$F = p A$

$F = p_y \, \delta x \, \delta z$

Pressure at a Point

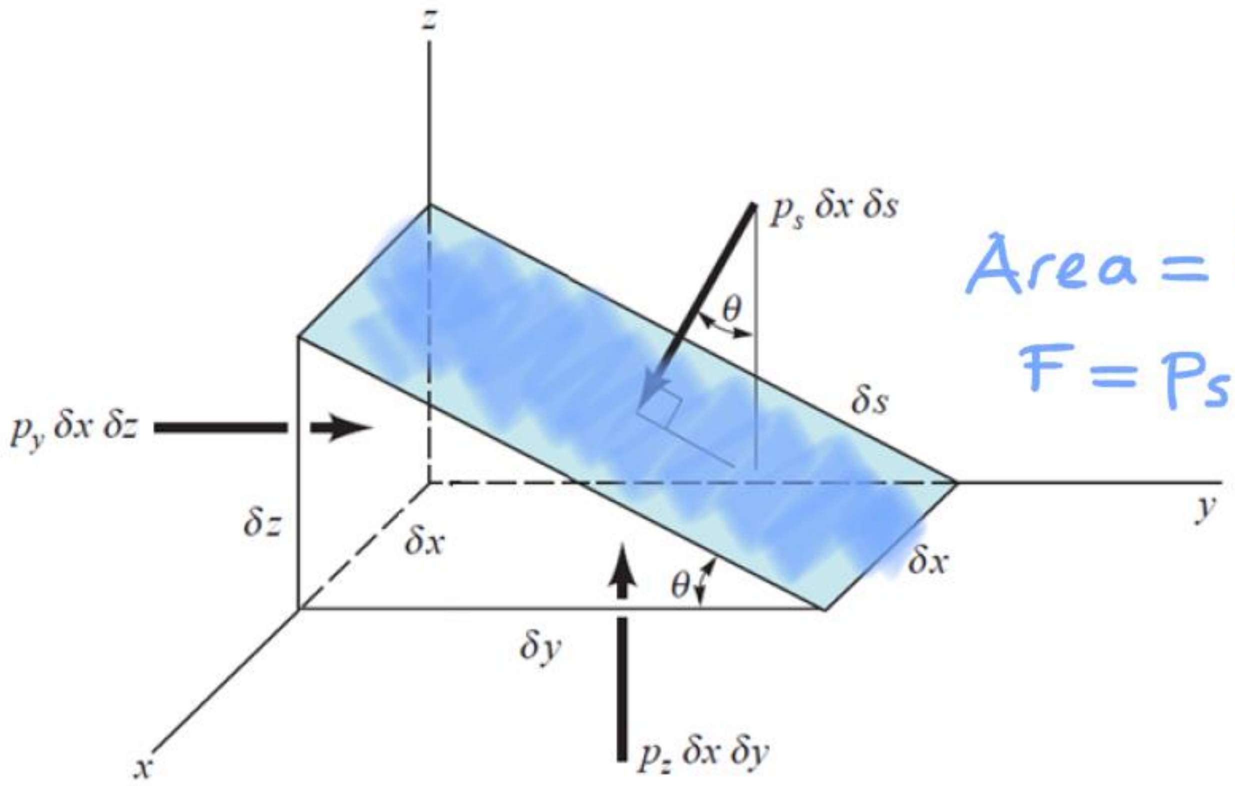


Area $= \delta x\, \delta y$

Force $=$ Pressure $\times$ Area

$F = P_z\, \delta x\, \delta y$

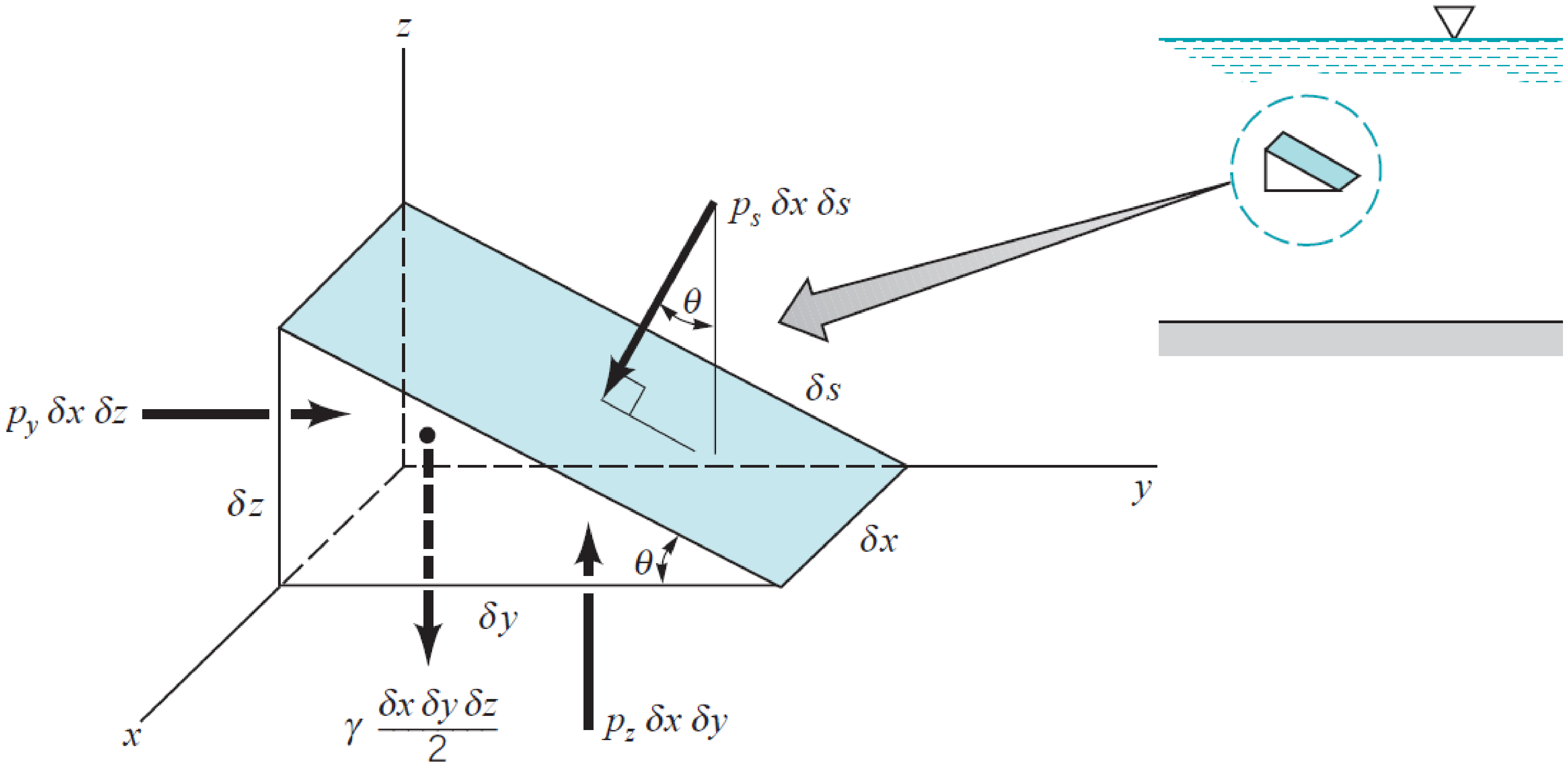
Pressure at a Point



Area $= \delta s\ \delta x$

$F = P_s\ \delta s\ \delta x$

Pressure at a Point



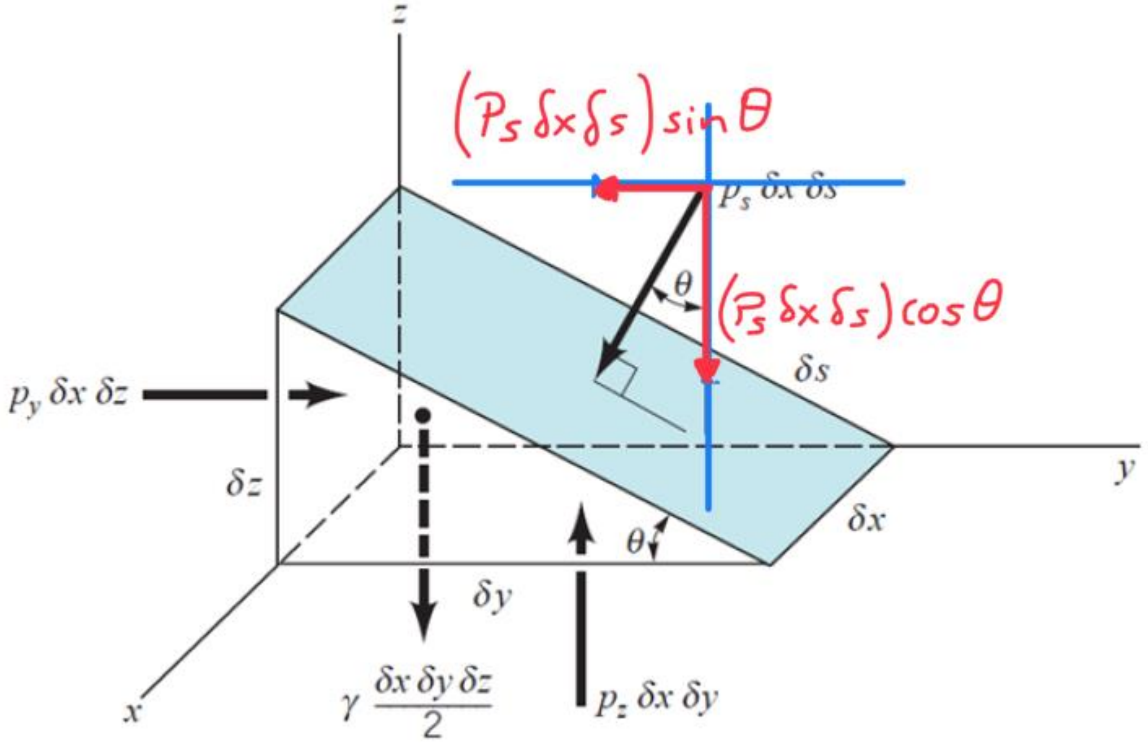
Forces on an arbitrary wedge-shaped element of fluid

Pressure at a Point

The equations of motion (Newton's second law, $F = ma$) in the y and z directions:

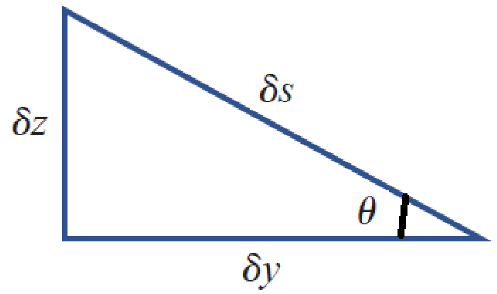
$$\sum F_y = p_y\,\delta x\,\delta z - \underline{p_s\,\delta x\,\delta s\,\sin\theta} = \rho\,\frac{\delta x\,\delta y\,\delta z}{2}\,a_y$$

$$\sum F_z = p_z\,\delta x\,\delta y - \underline{p_s\,\delta x\,\delta s\,\cos\theta} - \gamma\,\frac{\delta x\,\delta y\,\delta z}{2} = \rho\,\frac{\delta x\,\delta y\,\delta z}{2}\,a_z$$



Average pressures on the faces: p_s, p_y, and p_z
Fluid specific weight: γ
Fluid density: ρ
Accelerations: a_y, a_z

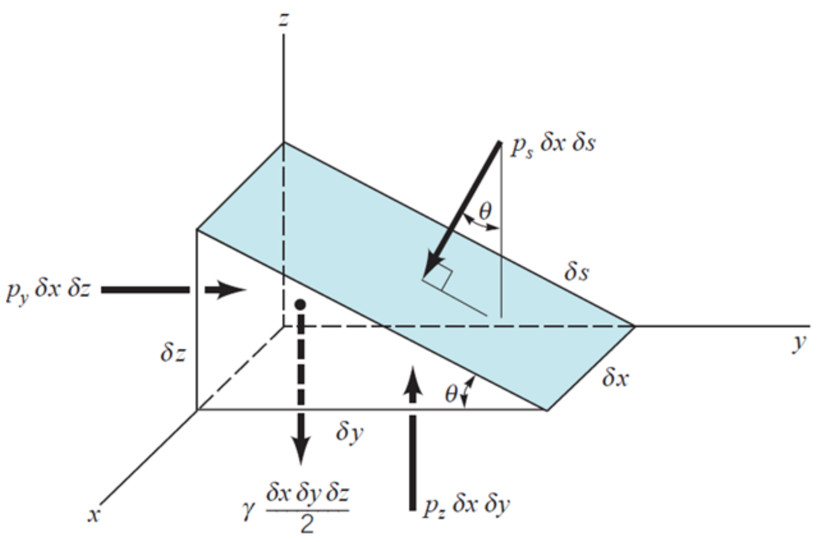
$$\delta y = \delta s\,\cos\theta$$
$$\delta z = \delta s\,\sin\theta$$



Pressure at a Point

$$\sum F_y = p_y\,\delta x\,\delta z - p_s\,\delta x\,\delta s\sin\theta = \rho\frac{\delta x\,\delta y\,\delta z}{2}a_y$$

$$\sum F_z = p_z\,\delta x\,\delta y - p_s\,\delta x\,\delta s\cos\theta - \gamma\frac{\delta x\,\delta y\,\delta z}{2} = \rho\frac{\delta x\,\delta y\,\delta z}{2}a_z$$



$$p_y - p_s = \rho a_y\frac{\delta y}{2}$$

$$p_z - p_s = (\rho a_z + \gamma)\frac{\delta z}{2}$$

Since we are really interested in what is happening at a point, we take the limit as δ_x, δ_y, and δ_z approach zero

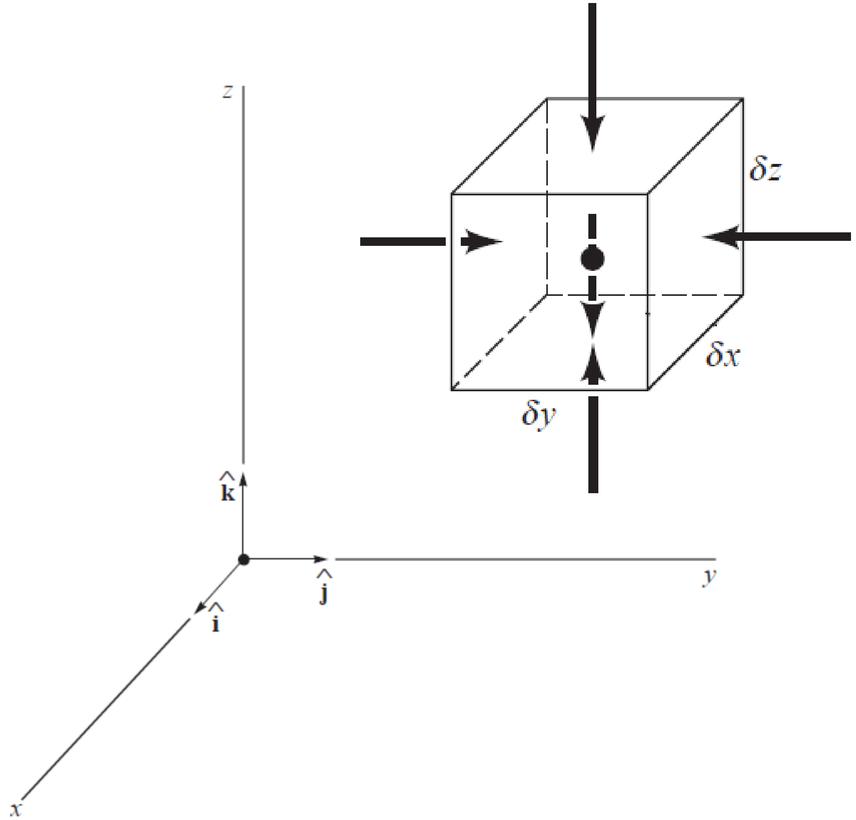
$$p_y = p_s \qquad p_z = p_s$$

$$\boxed{p_s = p_y = p_z}$$

The pressure at a point in a fluid at rest, or in motion, is independent of direction as long as there are no shearing stresses present. ***Pascal's law***

Basic Equation for Pressure Field

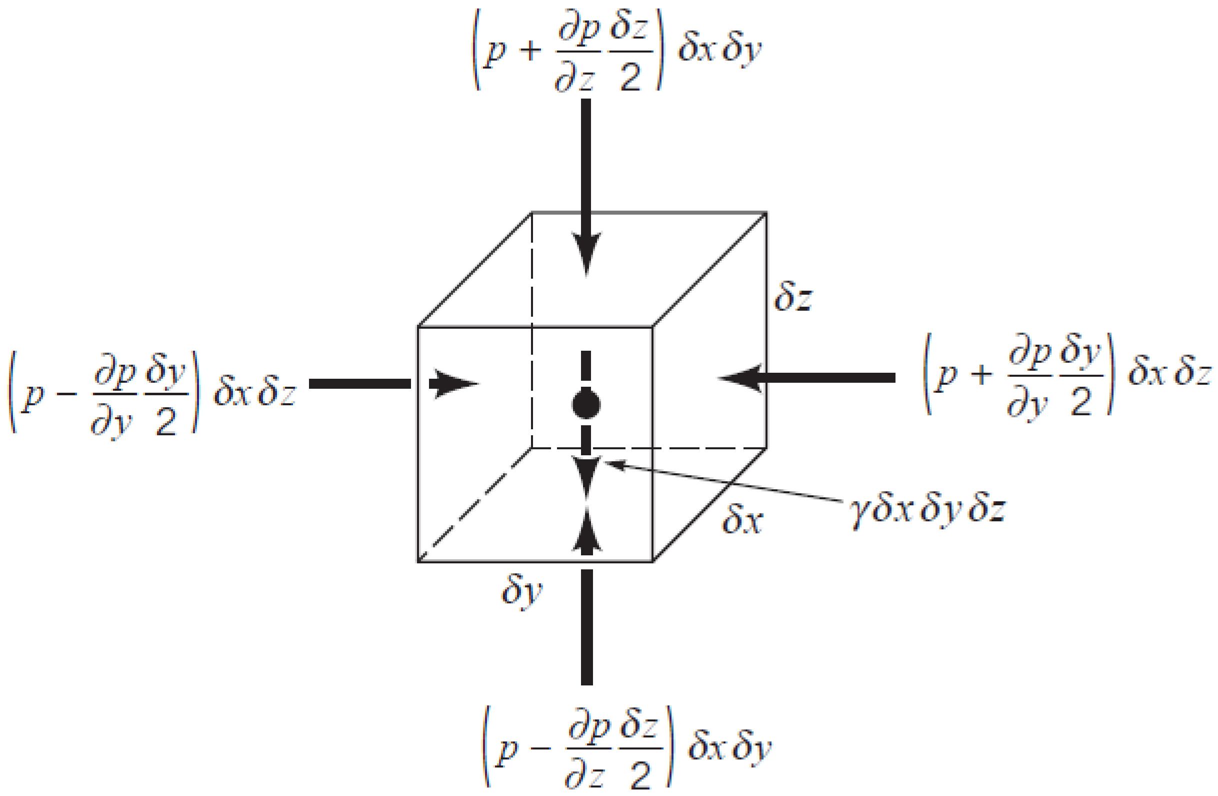
How does the pressure in a fluid in which there are no shearing stresses vary from point to point?



There are two types of forces acting on this element:
surface forces due to the pressure, and
a ***body force*** equal to the weight of the element.

Basic Equation for Pressure Field

There are two types of forces acting on this element: ***surface forces*** due to the pressure, and a ***body force*** equal to the weight of the element.

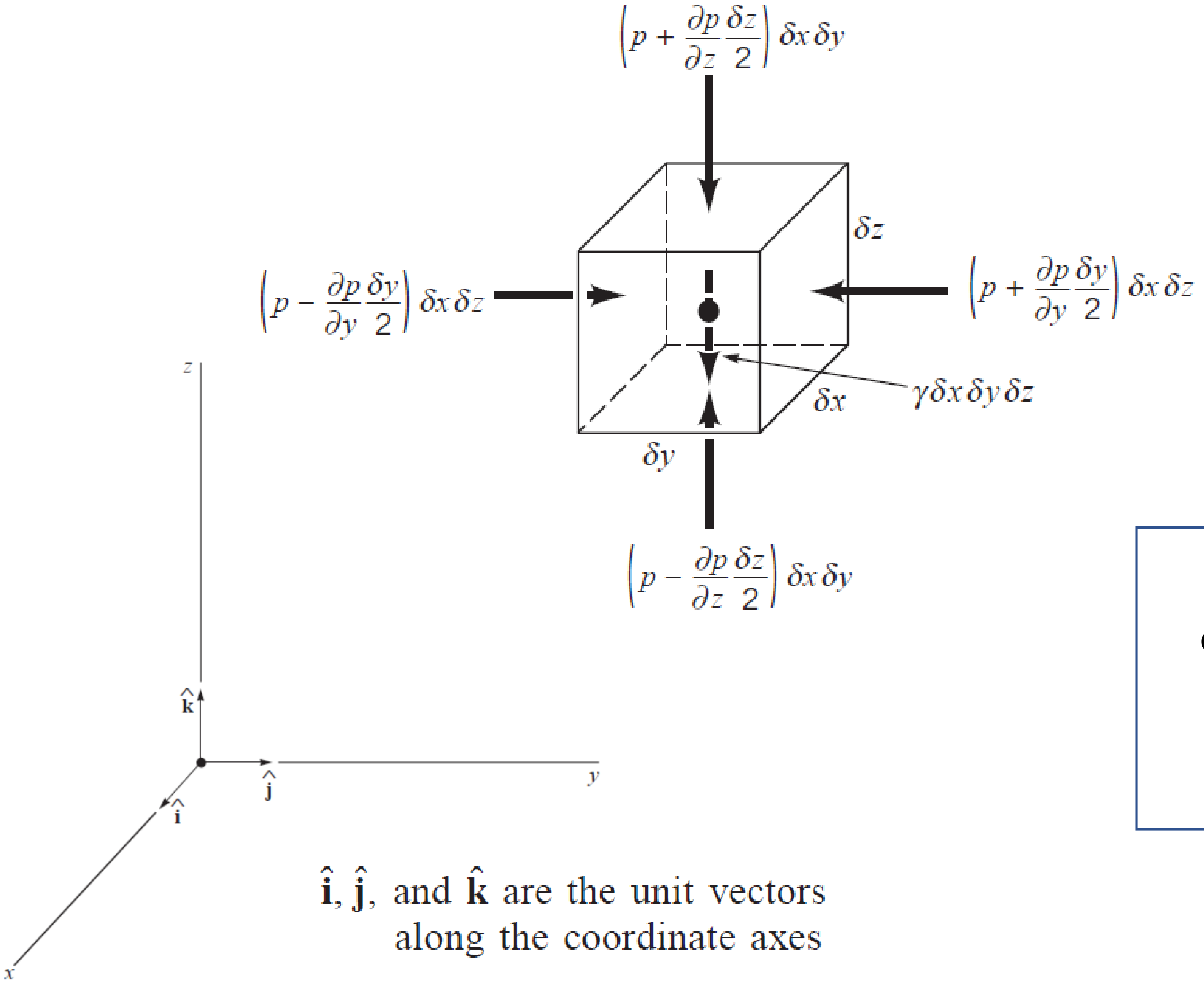


The resultant surface force in the y direction

$$\delta F_y = \left(p - \frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x\,\delta z - \left(p + \frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x\,\delta z$$

$$\delta F_y = -\frac{\partial p}{\partial y}\,\delta x\,\delta y\,\delta z$$

Basic Equation for Pressure Field



$\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are the unit vectors along the coordinate axes

resultant surface forces

$$\delta F_y = -\frac{\partial p}{\partial y}\,\delta x\,\delta y\,\delta z$$

$$\delta F_x = -\frac{\partial p}{\partial x}\,\delta x\,\delta y\,\delta z$$

$$\delta F_z = -\frac{\partial p}{\partial z}\,\delta x\,\delta y\,\delta z$$

The resultant surface force acting on the element can be expressed in vector form as

$$\delta \mathbf{F}_s = \delta F_x \hat{\mathbf{i}} + \delta F_y \hat{\mathbf{j}} + \delta F_z \hat{\mathbf{k}}$$

Basic Equation for Pressure Field

$$\delta \mathbf{F}_s = \delta F_x \hat{\mathbf{i}} + \delta F_y \hat{\mathbf{j}} + \delta F_z \hat{\mathbf{k}}$$

$$\delta F_y = -\frac{\partial p}{\partial y}\,\delta x\,\delta y\,\delta z$$

$$\delta F_x = -\frac{\partial p}{\partial x}\,\delta x\,\delta y\,\delta z$$

$$\delta F_z = -\frac{\partial p}{\partial z}\,\delta x\,\delta y\,\delta z$$

$$\delta \mathbf{F}_s = -\left(\frac{\partial p}{\partial x}\hat{\mathbf{i}} + \frac{\partial p}{\partial y}\hat{\mathbf{j}} + \frac{\partial p}{\partial z}\hat{\mathbf{k}}\right)\delta x\,\delta y\,\delta z$$

$$\nabla(\ ) = \frac{\partial(\ )}{\partial x}\hat{\mathbf{i}} + \frac{\partial(\ )}{\partial y}\hat{\mathbf{j}} + \frac{\partial(\ )}{\partial z}\hat{\mathbf{k}}$$

symbol ∇ is the *gradient* or "del" vector operator

$$\frac{\partial p}{\partial x}\hat{\mathbf{i}} + \frac{\partial p}{\partial y}\hat{\mathbf{j}} + \frac{\partial p}{\partial z}\hat{\mathbf{k}} = \nabla p$$

$$\frac{\delta \mathbf{F}_s}{\delta x\,\delta y\,\delta z} = -\nabla p$$

resultant surface force per unit volume

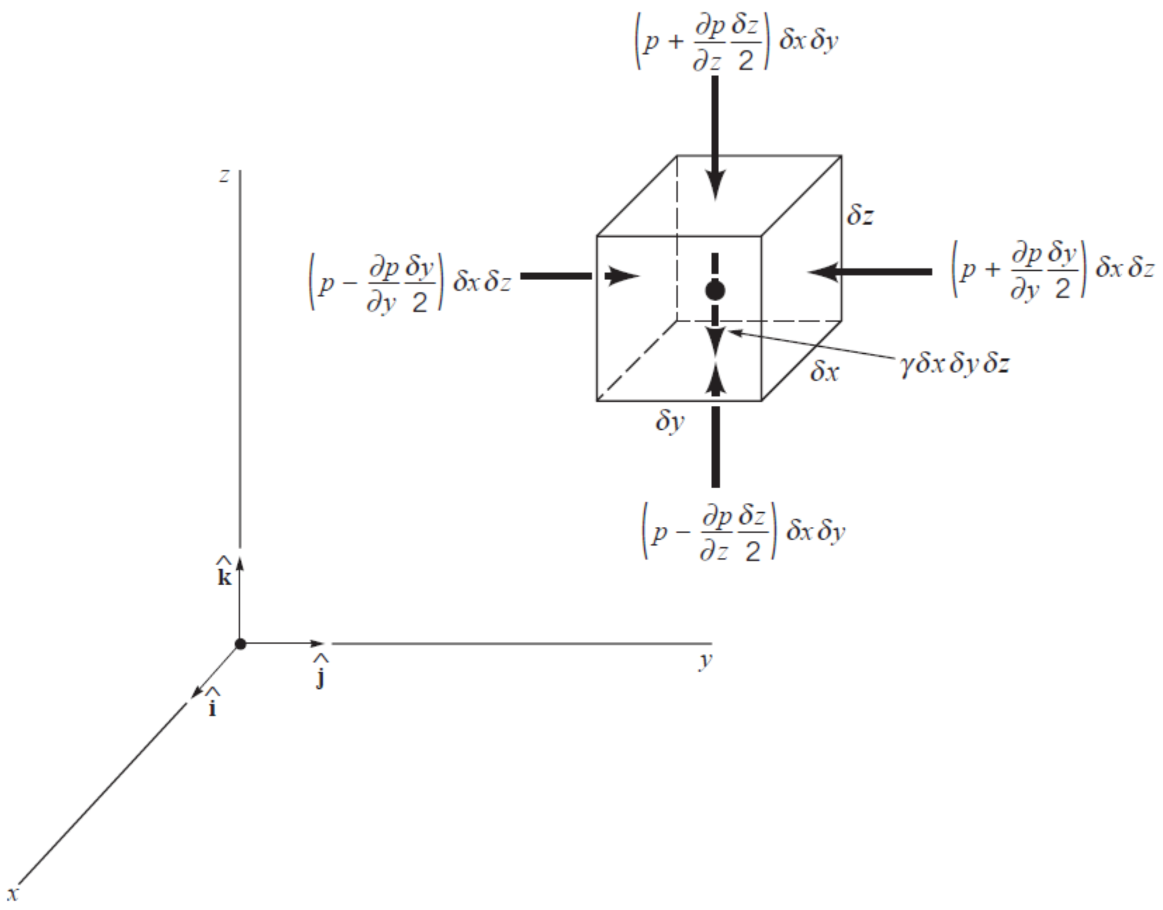
Basic Equation for Pressure Field

$$\boxed{\frac{\delta \mathbf{F}_s}{\delta x\, \delta y\, \delta z} = -\nabla p}$$

resultant surface force per unit volume

$$\boxed{-\delta \mathscr{W} \hat{\mathbf{k}} = -\gamma\, \delta x\, \delta y\, \delta z\, \hat{\mathbf{k}}}$$

z axis is vertical,
the weight of the element



Newton's second law, applied to the fluid element

$$\sum \delta \mathbf{F} = \delta m\, \mathbf{a}$$

$\Sigma\, \delta \mathbf{F}$ represents the resultant force acting on the element

$$\sum \delta \mathbf{F} = \delta \mathbf{F}_s - \delta \mathscr{W} \hat{\mathbf{k}}$$
$$= -\nabla p\, \delta x\, \delta y\, \delta z - \gamma\, \delta x\, \delta y\, \delta z\, \hat{\mathbf{k}}$$

$\mathbf{a}$ is the acceleration of the element

δm is the element mass, $\rho\, \delta x\, \delta y\, \delta z$

$$\delta m\, \mathbf{a} = \rho\, \delta x\, \delta y\, \delta z\, \mathbf{a}$$

$$\boxed{-\nabla p - \gamma \hat{\mathbf{k}} = \rho \mathbf{a}}$$

Pressure Variation in a Fluid at Rest

$$\boxed{-\nabla p - \gamma \hat{\mathbf{k}} = \rho \mathbf{a}}$$

general equation of motion for a fluid in which there are no shearing stresses

For a fluid at rest $\mathbf{a} = 0$

$$\nabla p + \gamma \hat{\mathbf{k}} = 0 \qquad \nabla p = -\gamma \hat{\mathbf{k}}$$

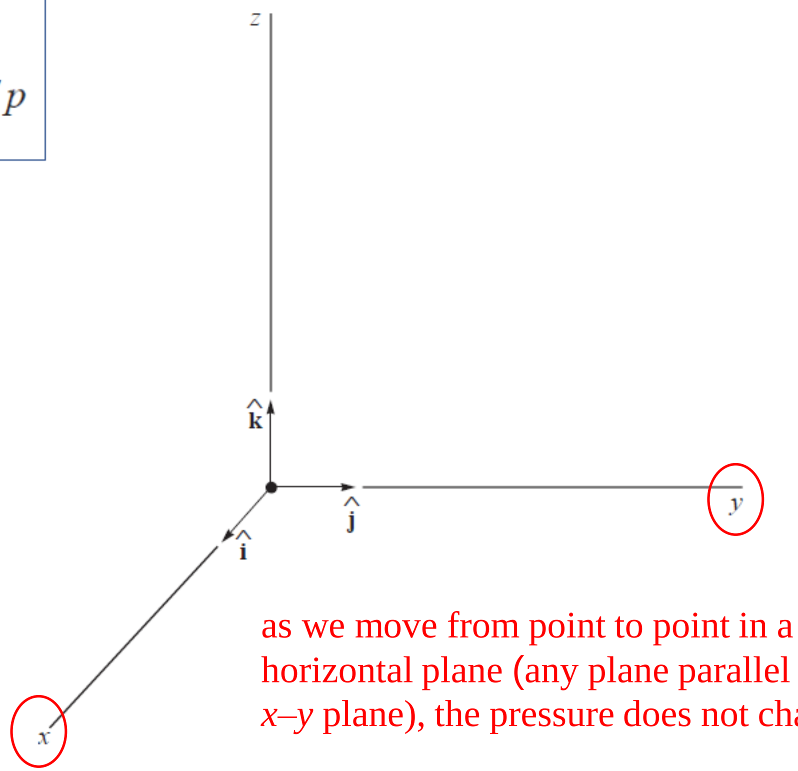
$$\frac{\partial p}{\partial x}\hat{\mathbf{i}} + \frac{\partial p}{\partial y}\hat{\mathbf{j}} + \frac{\partial p}{\partial z}\hat{\mathbf{k}} = \nabla p = -\gamma \hat{\mathbf{k}}$$

$$\underbrace{\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = 0}_{\text{pressure does not depend on } x \text{ or } y} \qquad \frac{\partial p}{\partial z} = -\gamma$$

$$\nabla(\ ) = \frac{\partial(\ )}{\partial x}\hat{\mathbf{i}} + \frac{\partial(\ )}{\partial y}\hat{\mathbf{j}} + \frac{\partial(\ )}{\partial z}\hat{\mathbf{k}}$$

symbol ∇ is the *gradient* or "del" vector operator

$$\frac{\partial p}{\partial x}\hat{\mathbf{i}} + \frac{\partial p}{\partial y}\hat{\mathbf{j}} + \frac{\partial p}{\partial z}\hat{\mathbf{k}} = \nabla p$$



as we move from point to point in a horizontal plane (any plane parallel to the *x*–*y* plane), the pressure does not change

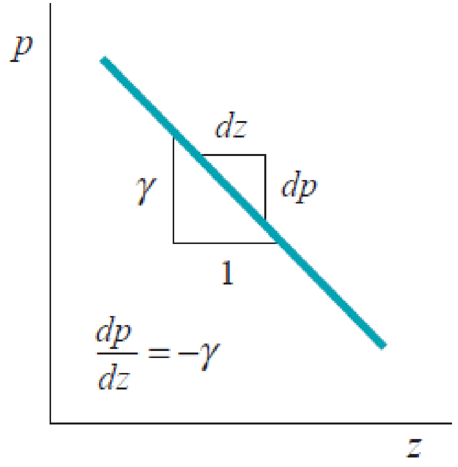
Pressure Variation in a Fluid at Rest

$$\underbrace{\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = 0}_{\text{pressure does not depend on } x \text{ or } y} \qquad \frac{\partial p}{\partial z} = -\gamma$$

Since p depends only on z, equation can be written as the ordinary differential equation

$$\boxed{\frac{dp}{dz} = -\gamma}$$

pressure decreases as we move upward in a fluid at rest



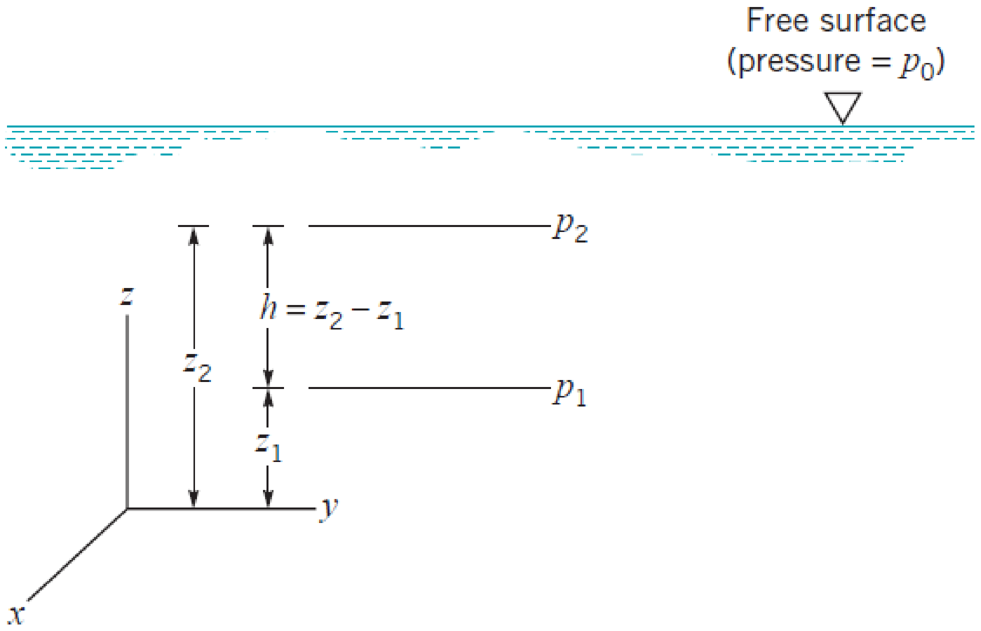
Pressure Variation in a Fluid at Rest – Incompressible Fluid

$$\boxed{\frac{dp}{dz} = -\gamma}$$

$$\int_{p_1}^{p_2} dp = -\gamma \int_{z_1}^{z_2} dz$$

$$p_1 - p_2 = \gamma h$$

$$\boxed{p_1 = \gamma h + p_2}$$



This type of pressure distribution is commonly called **hydrostatic distribution**.

in an incompressible fluid at rest the pressure varies linearly with depth

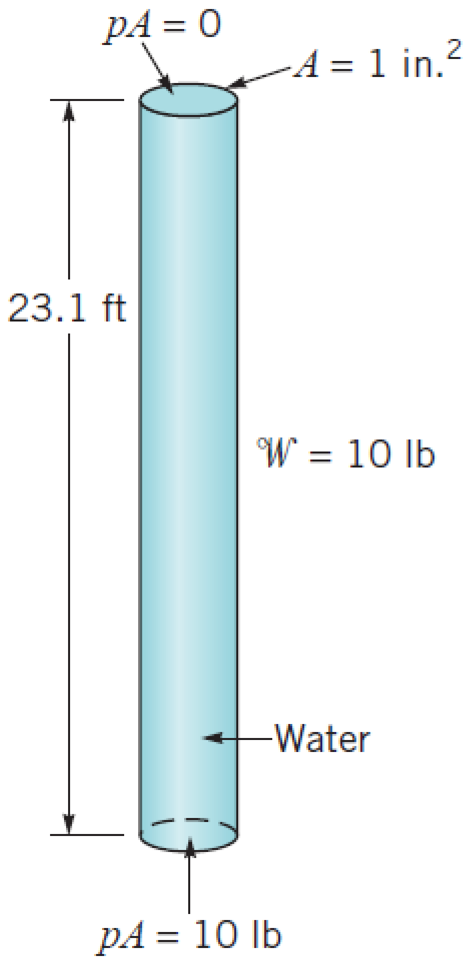
Pressure Variation in a Fluid at Rest – Incompressible Fluid

PRESSURE HEAD

$$\boxed{p_1 = \gamma h + p_2}$$

$$h = \frac{p_1 - p_2}{\gamma}$$

h is called the ***pressure head*** and is interpreted as the height of a column of fluid of specific weight γ required to give a pressure difference $p_1 - p_2$.



Pressure difference 10 psi
Express as pressure head:

pressure head as 23.1 ft of water
(γ = 62.4 lb/ft^3)

518 mm of Hg
(γ = 133 kN/m^3)

Pressure Variation in a Fluid at Rest – Incompressible Fluid

PRESSURE HEAD

Pressure difference : $10 \dfrac{lb}{in^2}$ (10 psi)

? ft of water

? mm of Hg

$$\text{Pressure head} = \frac{\text{Pressure difference}}{\text{Specific weight of liquid used in "pressure head" expression}}$$

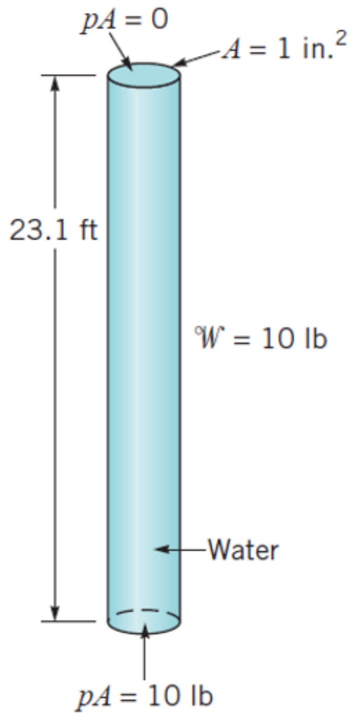
Pressure Variation in a Fluid at Rest – Incompressible Fluid

PRESSURE HEAD

$$h_{H_2O} = \frac{\Delta P}{\gamma_{H_2O}} = \frac{10 \, \frac{\cancel{lb}}{\cancel{in^2}} \left[\frac{12^2 \, \cancel{in^2}}{1 \, ft^2}\right]}{62.4 \, \frac{\cancel{lb}}{ft^{\cancel{3}}}}$$

$$= \frac{10 * 144}{62.4} = 23.1 \text{ ft of water}$$

$\boxed{12 \text{ in} = 1 \text{ ft}}$



Pressure difference 10 psi
Express as pressure head:

pressure head as 23.1 ft of water ($\gamma = 62.4$ lb/ft^3)

518 mm of Hg ($\gamma = 133$ kN/m^3)

Pressure Variation in a Fluid at Rest – Incompressible Fluid

PRESSURE HEAD

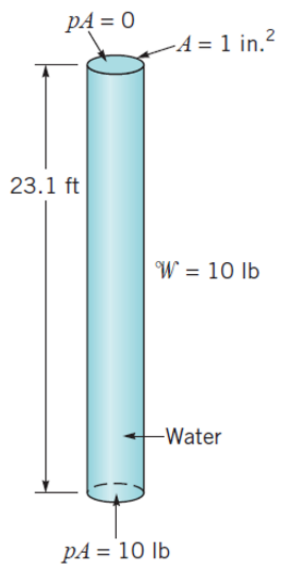
$$h_{Hg} = \frac{\Delta P}{\gamma_{Hg}} = \frac{10\,\frac{\cancel{lb}}{\cancel{in^2}}\left[\frac{12\,\cancel{in}}{1\,\cancel{ft}}\right]^2 \frac{4.4482\,\cancel{N}}{1\,\cancel{lb}}\cdot\frac{1\,\cancel{kN}}{1000\,\cancel{N}}\left[\frac{1\,\cancel{ft}}{0.3048\,\cancel{m}}\right]^2}{133\,\frac{\cancel{kN}}{m^{\cancel{3}}}}$$

$$\frac{10 * 12^2 * 4.4482 * \frac{1}{1000} * \frac{1}{0.3048^2}}{133} = 0.518\ m$$

of mercury

$$0.518\ m\ \frac{1000\ mm}{1\ m} = 518\ mm$$

Hg



Pressure difference 10 psi
Express as pressure head:

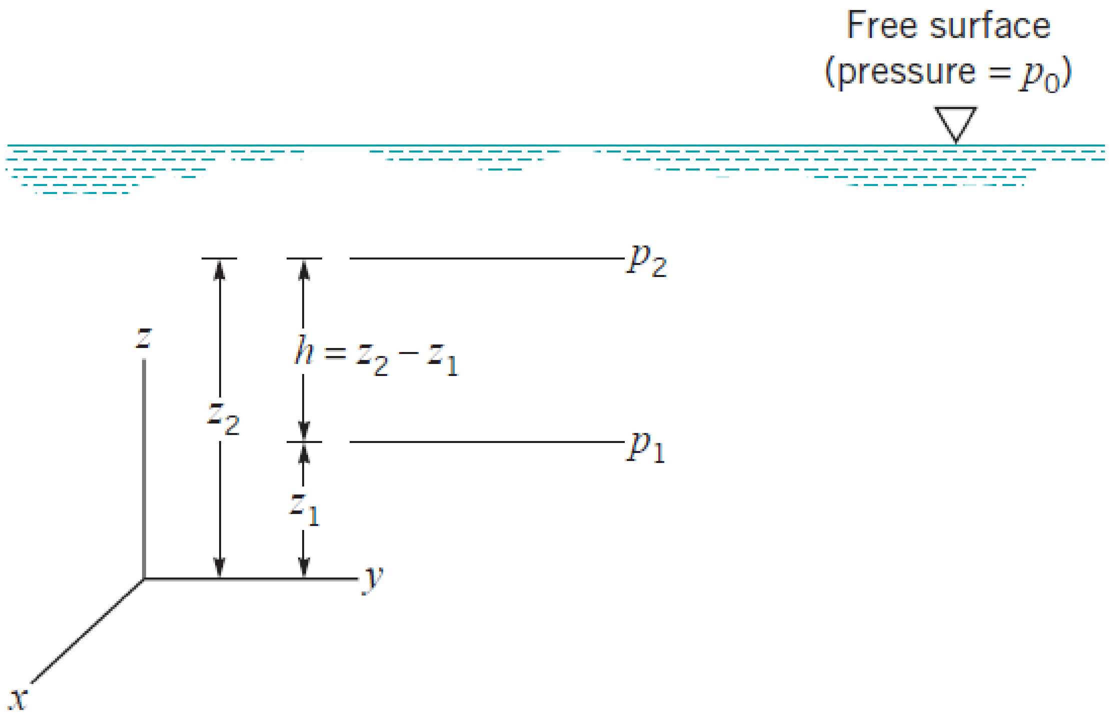
pressure head as 23.1 ft of water
(γ = 62.4 lb/ft³)

518 mm of Hg
(γ = 133 kN/m³)

12 in = 1 ft
1 ft = 0.3048 m
1 lb = 4.4482 N

FLUID STATICS - Pressure Variation in a Fluid at Rest

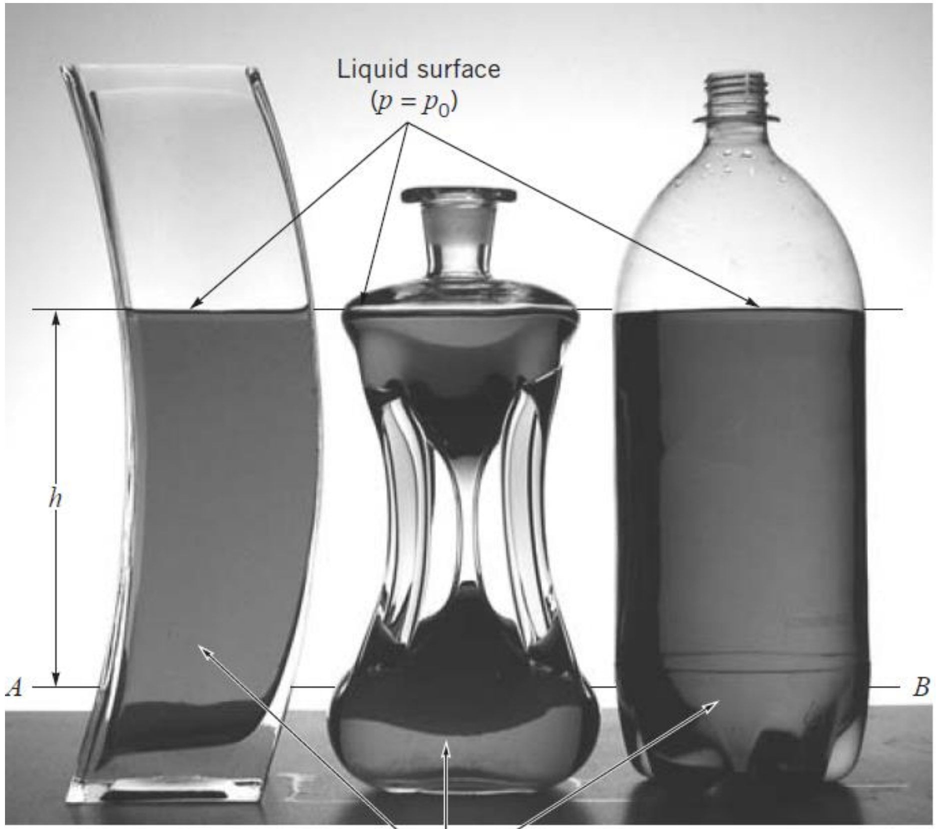
Incompressible Fluid

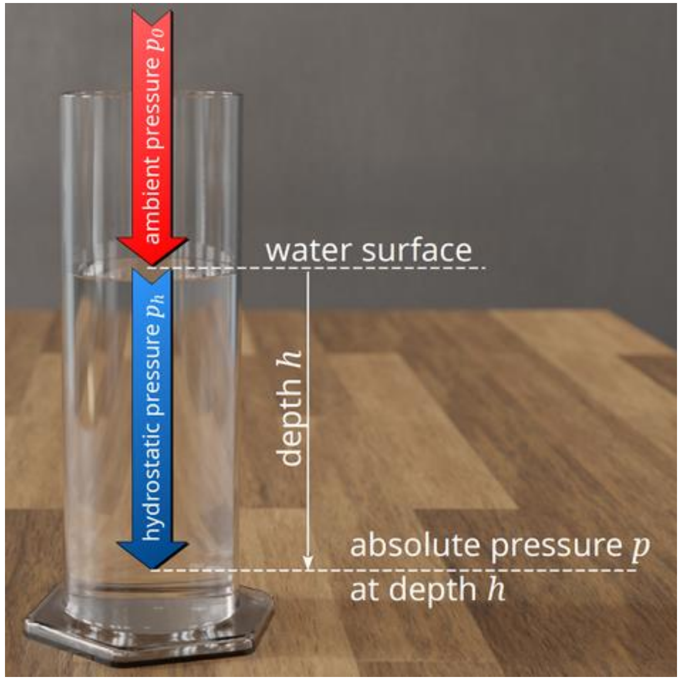


atmospheric pressure

FLUID STATICS - Pressure Variation in a Fluid at Rest

Incompressible Fluid

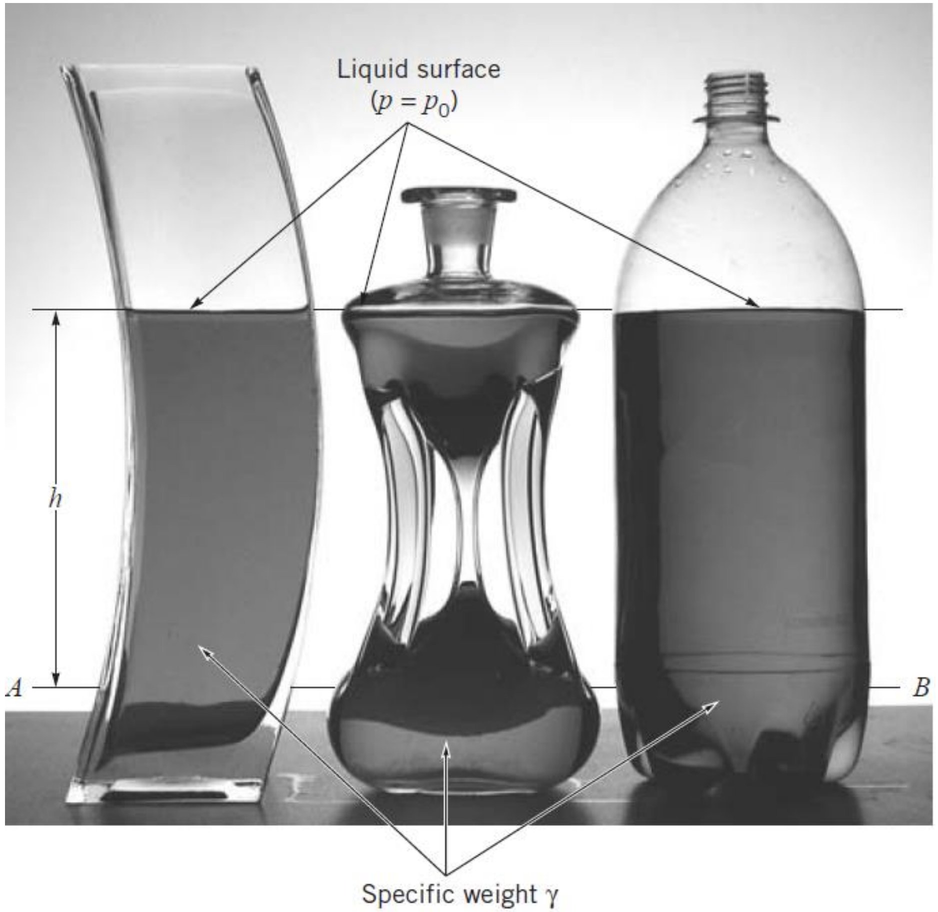


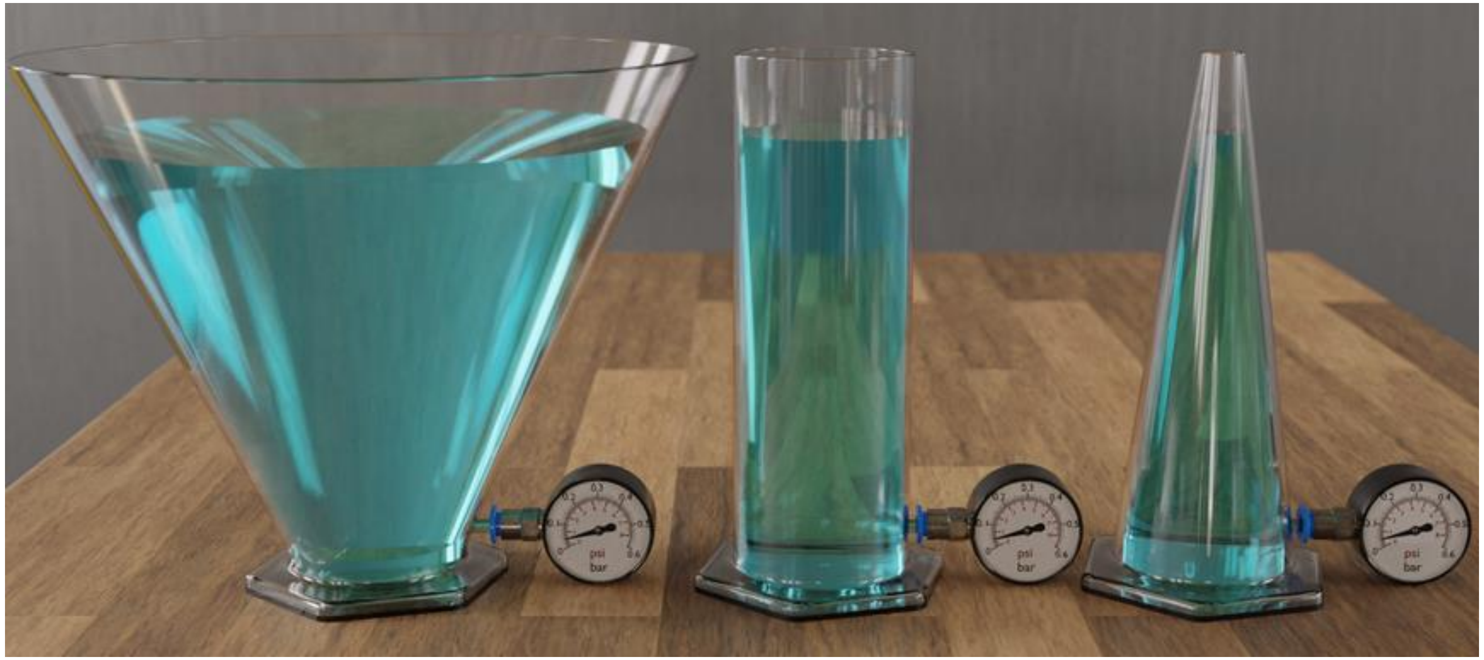


$$p = \gamma h + p_0$$

FLUID STATICS - Pressure Variation in a Fluid at Rest

Incompressible Fluid





$$p = \gamma h + p_0$$

FLUID STATICS - Pressure Variation in a Fluid at Rest

Incompressible Fluid

EXAMPLE 2.1 Pressure–Depth Relationship

GIVEN Because of a leak in a buried gasoline storage tank, water has seeped in to the depth shown in Fig. E2.1. The specific gravity of the gasoline is $SG = 0.68$.

FIND Determine the pressure at the gasoline–water interface and at the bottom of the tank. Express the pressure in units of lb/ft^2, lb/in.^2, and as a pressure head in feet of water.

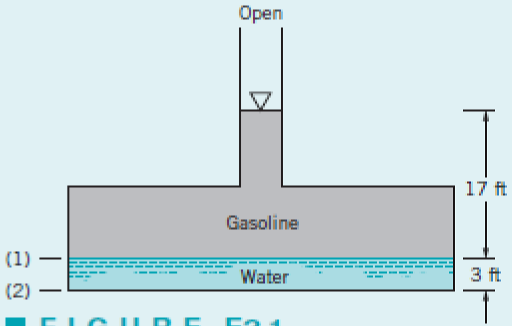


FIGURE E2.1

SOLUTION

Since we are dealing with liquids at rest, the pressure distribution will be hydrostatic, and therefore the pressure variation can be found from the equation:

$$p = \gamma h + p_0$$

With p_0 corresponding to the pressure at the free surface of the gasoline, then the pressure at the interface is

$$\begin{aligned} p_1 &= SG\gamma_{H_2O} h + p_0 \\ &= (0.68)(62.4 \text{ lb/ft}^3)(17 \text{ ft}) + p_0 \\ &= 721 + p_0 \ (\text{lb/ft}^2) \end{aligned}$$

If we measure the pressure relative to atmospheric pressure (gage pressure), it follows that $p_0 = 0$, and therefore

$$p_1 = 721 \text{ lb/ft}^2 \qquad \text{(Ans)}$$

$$p_1 = \frac{721 \text{ lb/ft}^2}{144 \text{ in.}^2/\text{ft}^2} = 5.01 \text{ lb/in.}^2 \qquad \text{(Ans)}$$

$$\frac{p_1}{\gamma_{H_2O}} = \frac{721 \text{ lb/ft}^2}{62.4 \text{ lb/ft}^3} = 11.6 \text{ ft} \qquad \text{(Ans)}$$

It is noted that a rectangular column of water 11.6 ft tall and 1 ft^2 in cross section weighs 721 lb. A similar column with a 1-in.^2 cross section weighs 5.01 lb.

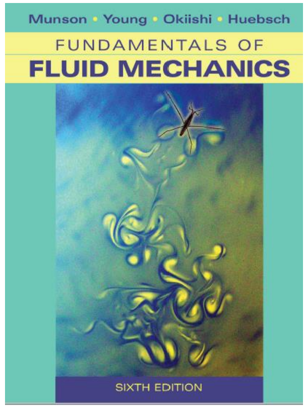
We can now apply the same relationship to determine the pressure at the tank bottom; that is,

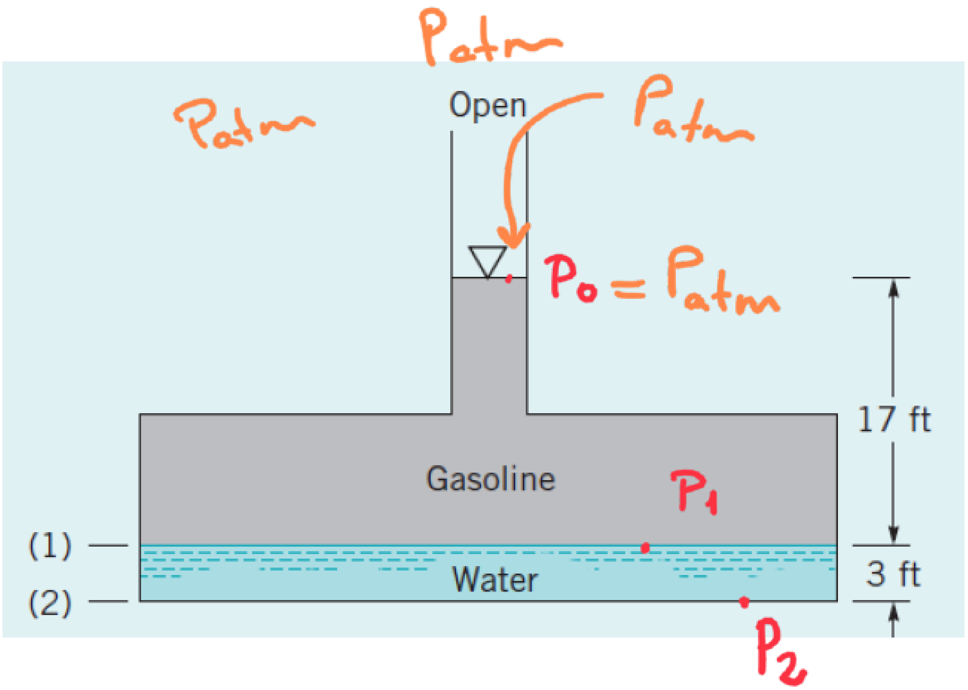
$$\begin{aligned} p_2 &= \gamma_{H_2O} h_{H_2O} + p_1 \\ &= (62.4 \text{ lb/ft}^3)(3 \text{ ft}) + 721 \text{ lb/ft}^2 \qquad \text{(Ans)} \\ &= 908 \text{ lb/ft}^2 \end{aligned}$$

$$p_2 = \frac{908 \text{ lb/ft}^2}{144 \text{ in.}^2/\text{ft}^2} = 6.31 \text{ lb/in.}^2 \qquad \text{(Ans)}$$

$$\frac{p_2}{\gamma_{H_2O}} = \frac{908 \text{ lb/ft}^2}{62.4 \text{ lb/ft}^3} = 14.6 \text{ ft} \qquad \text{(Ans)}$$

COMMENT Observe that if we wish to express these pressures in terms of *absolute* pressure, we would have to add the local atmospheric pressure (in appropriate units) to the previous results. A further discussion of gage and absolute pressure is given in Section 2.5.





$$P_1 = P_o + \gamma_g h_g$$

$$P_2 = P_1 + \gamma_{H_2O} h_{H_2O} \quad \text{or}$$

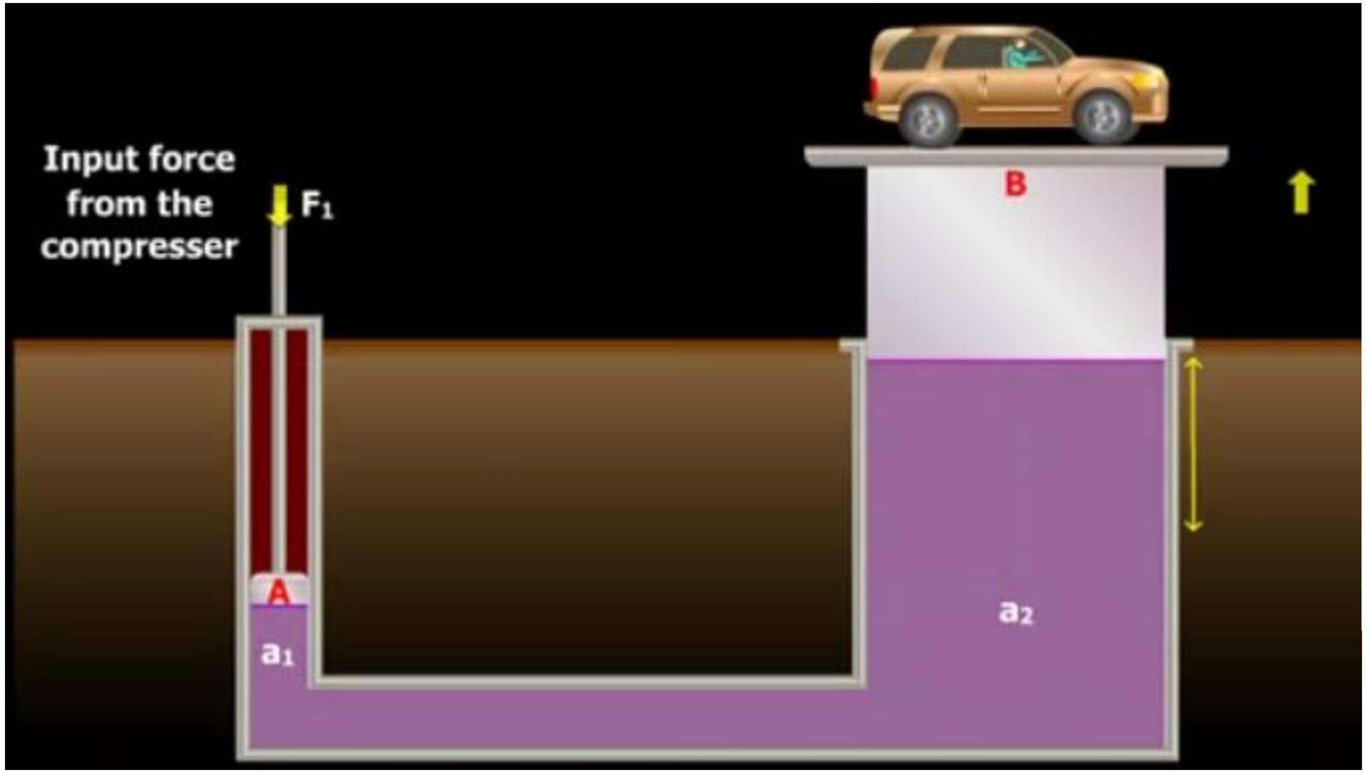
$$P_2 = P_o + \gamma_g h_g + \gamma_{H_2O} h_{H_2O}$$

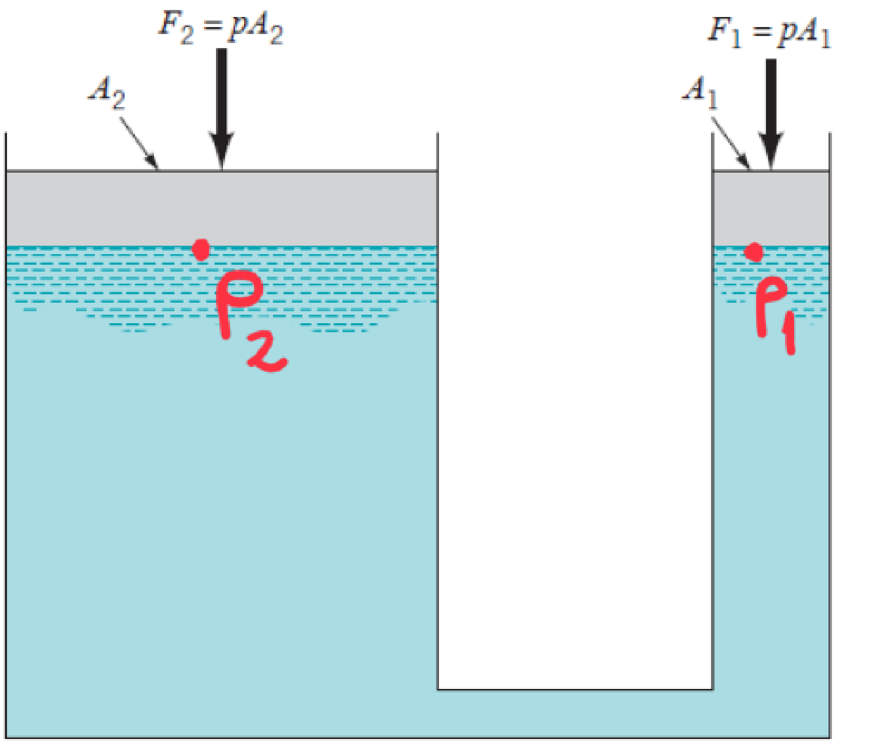
$P_{atm} = 0$ (gage)

FLUID STATICS - Pressure Variation in a Fluid at Rest

Incompressible Fluid

Transmission of Fluid Pressure





p acting on the faces of both pistons is the same

$P_1 = P_2 = P$

$$F_2 = (A_2/A_1)F_1$$

FLUID STATICS - Pressure Variation in a Fluid at Rest

Incompressible Fluid

Transmission of Fluid Pressure

Example 2.1

The diameters of cylindrical pistons A and B are 3 cm and 20 cm, respectively. The faces of the pistons are at the same elevation, and the intervening passages are filled with an incompressible hydraulic oil. A force P of 100 N is applied at the end of the lever, as shown in Figure 2.3. What weight W can the hydraulic jack support?

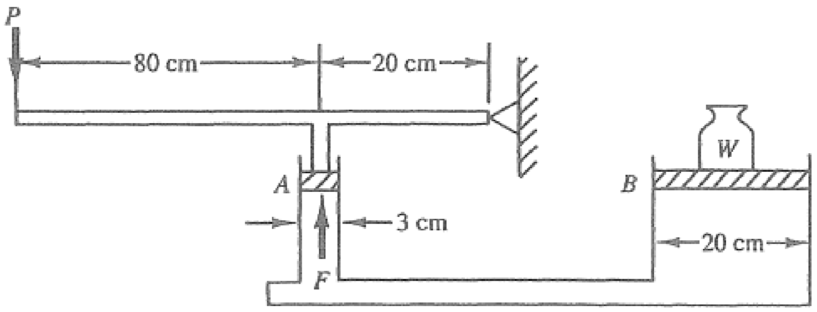


Figure 2.3 Hydraulic jack

Solution

Balancing the moments produced by P and F around the pin connection yields

$$(100\text{ N})(100\text{ cm}) = F(20\text{ cm})$$

Thus,

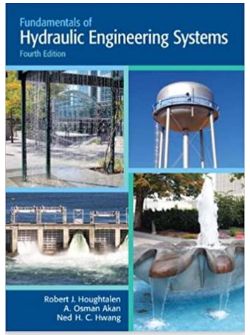
$$F = 500\text{ N}$$

From Pascal's law, the pressure P_A applied at A is the same as that of P_B applied at B. Therefore,

$$P_A = \frac{F}{[(\pi \cdot 3^2)/4]\text{ cm}^2} \qquad P_B = \frac{W}{[(\pi \cdot 20^2)/4]\text{ cm}^2}$$

$$\frac{500\text{ N}}{7.07\text{ cm}^2} = \frac{W}{314\text{ cm}^2}$$

$$\therefore W = 500\text{ N}\left(\frac{314\text{ cm}^2}{7.07\text{ cm}^2}\right) = 2.22 \times 10^4\text{ N}$$



FLUID STATICS - Pressure Variation in a Fluid at Rest

Compressible Fluid

Gases such are compressible fluids since the density of the gas can change significantly with changes in pressure & temperature.

$$\boxed{\frac{dp}{dz} = -\gamma}$$

FLUID STATICS - Pressure Variation in a Fluid at Rest

Compressible Fluid

Ideal gas law $$\rho = \frac{p}{RT}$$ $$(\gamma = \rho g)$$

$$\rho g = \frac{gp}{RT}$$

$$\rho g = \frac{gp}{RT}$$

$$-\gamma = -\frac{gp}{RT}$$

$$\boxed{\frac{dp}{dz} = -\gamma}$$

$$\boxed{\frac{dp}{dz} = -\frac{gp}{RT}}$$

FLUID STATICS - Pressure Variation in a Fluid at Rest

Compressible Fluid

$$\frac{dp}{dz} = -\frac{gp}{RT}$$

$$\int_{p_1}^{p_2} \frac{dp}{p} = \ln \frac{p_2}{p_1} = -\frac{g}{R} \int_{z_1}^{z_2} \frac{dz}{T}$$

$$\boxed{p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0} \right]}$$

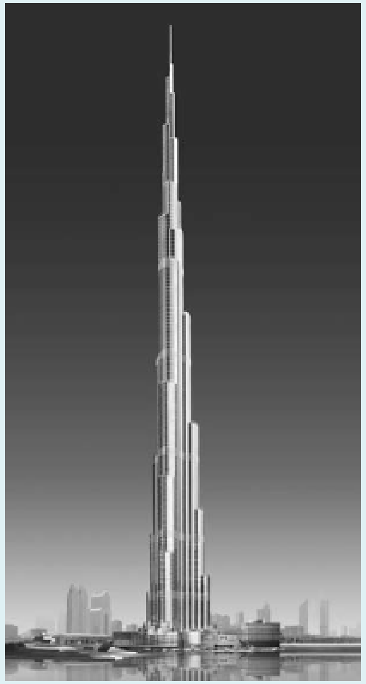
FLUID STATICS - Pressure Variation in a Fluid at Rest

Compressible Fluid

EXAMPLE 2.2 Incompressible and Isothermal Pressure–Depth Variations

GIVEN In 2007 the Burj Dubai skyscraper being built in the United Arab Emirates reached the stage in its construction where it became the world's tallest building. When completed it is expected to be at least 2275 ft tall, although its final height remains a secret.

FIND (a) Estimate the ratio of the pressure at the projected 2275-ft top of the building to the pressure at its base, assuming the air to be at a common temperature of 59 °F. (b) Compare the pressure calculated in part (a) with that obtained by assuming the air to be incompressible with $\gamma = 0.0765 \text{ lb/ft}^3$ at 14.7 psi (abs) (values for air at standard sea level conditions).



■ FIGURE E2.2 (Figure courtesy of Emaar Properties, Dubai, UAE.)

SOLUTION

For the assumed isothermal conditions, and treating air as a compressible fluid, Eq. 2.10 can be applied to yield

$$\frac{p_2}{p_1} = \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

$$= \exp\left\{-\frac{(32.2 \text{ ft/s}^2)(2275 \text{ ft})}{(1716 \text{ ft}\cdot\text{lb/slug}\cdot{}^\circ\text{R})[(59 + 460)^\circ\text{R}]}\right\}$$

$$= 0.921 \qquad \text{(Ans)}$$

If the air is treated as an incompressible fluid we can apply Eq. 2.5. In this case

$$p_2 = p_1 - \gamma(z_2 - z_1)$$

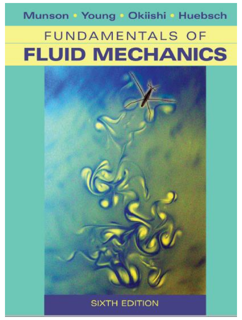
or

$$\frac{p_2}{p_1} = 1 - \frac{\gamma(z_2 - z_1)}{p_1}$$

$$= 1 - \frac{(0.0765 \text{ lb/ft}^3)(2275 \text{ ft})}{(14.7 \text{ lb/in.}^2)(144 \text{ in.}^2/\text{ft}^2)} = 0.918 \qquad \text{(Ans)}$$

COMMENTS Note that there is little difference between the two results. Since the pressure difference between the bottom and top of the building is small, it follows that the variation in fluid density is small and, therefore, the compressible fluid and incompressible fluid analyses yield essentially the same result.

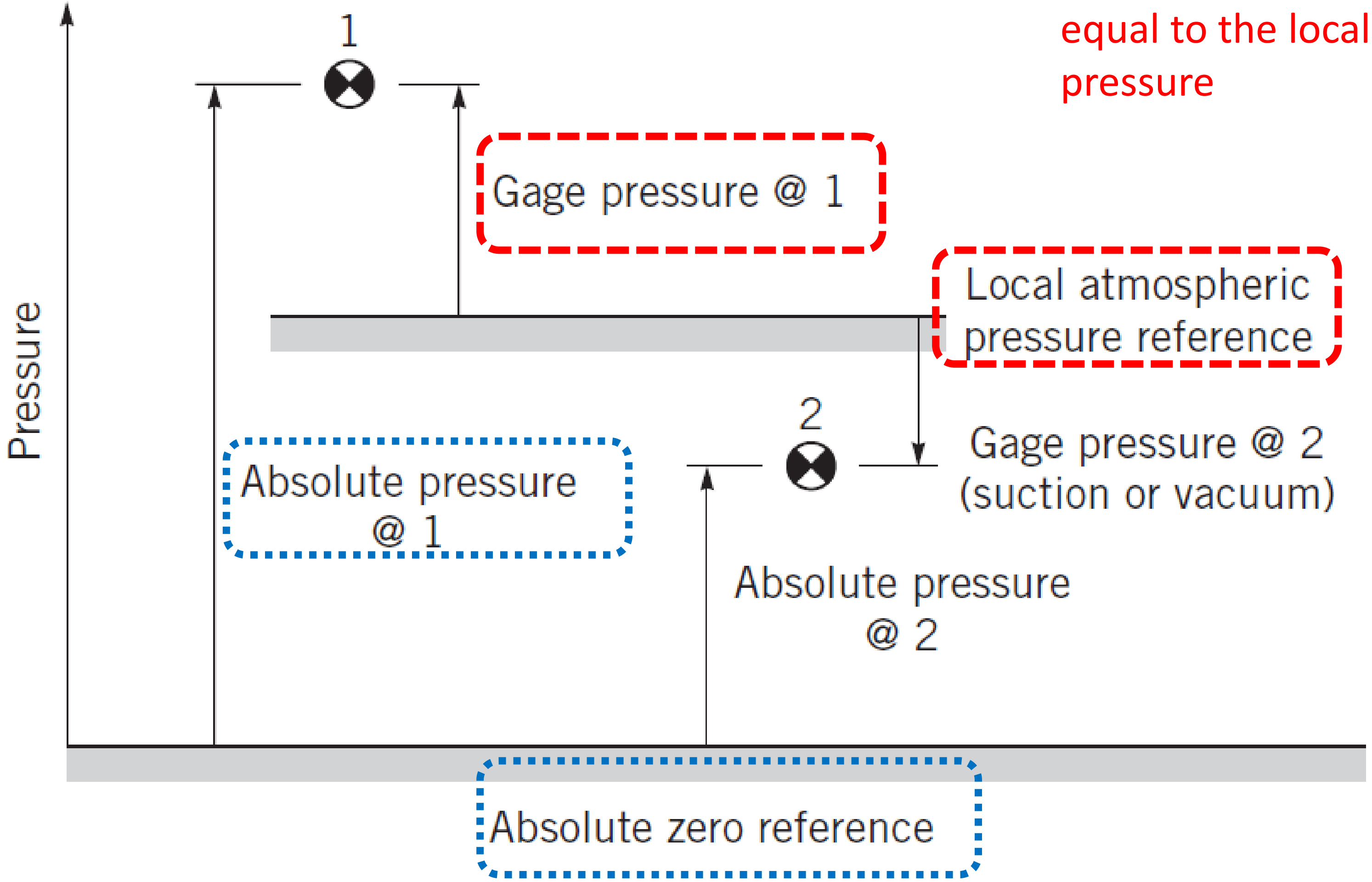
We see that for both calculations the pressure decreases by approximately 8% as we go from ground level to the top of this tallest building. It does not require a very large pressure difference to support a 2275-ft-tall column of fluid as light as air. This result supports the earlier statement that the changes in pressures in air and other gases due to elevation changes are very small, even for distances of hundreds of feet. Thus, the pressure differences between the top and bottom of a horizontal pipe carrying a gas, or in a gas storage tank, are negligible since the distances involved are very small.



FLUID STATICS – Measurement of Pressure

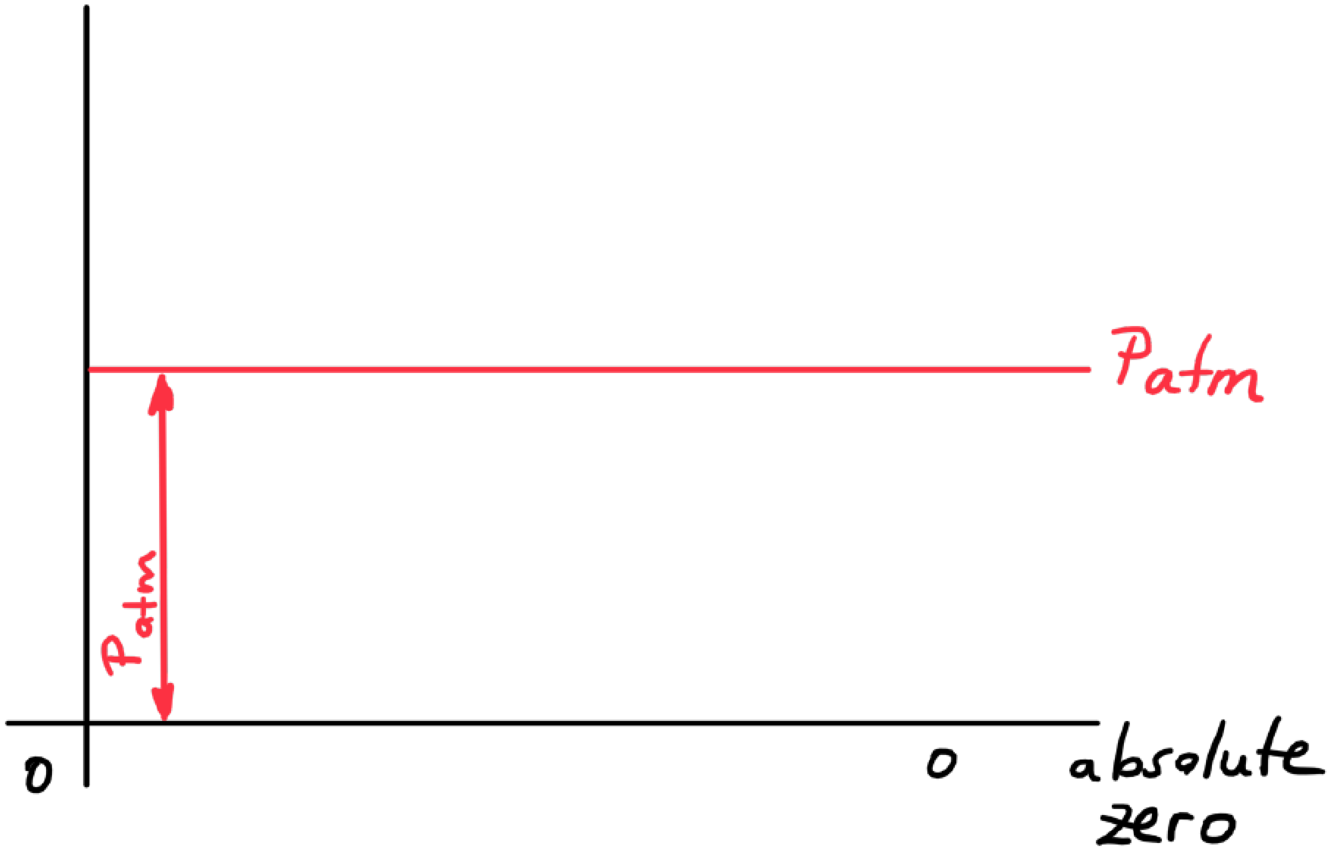
Absolute Pressure & Gage Pressure

Gage pressure is measured relative to the local atmospheric pressure. Thus a gage pressure of zero corresponds to a pressure that is equal to the local atmosperic pressure

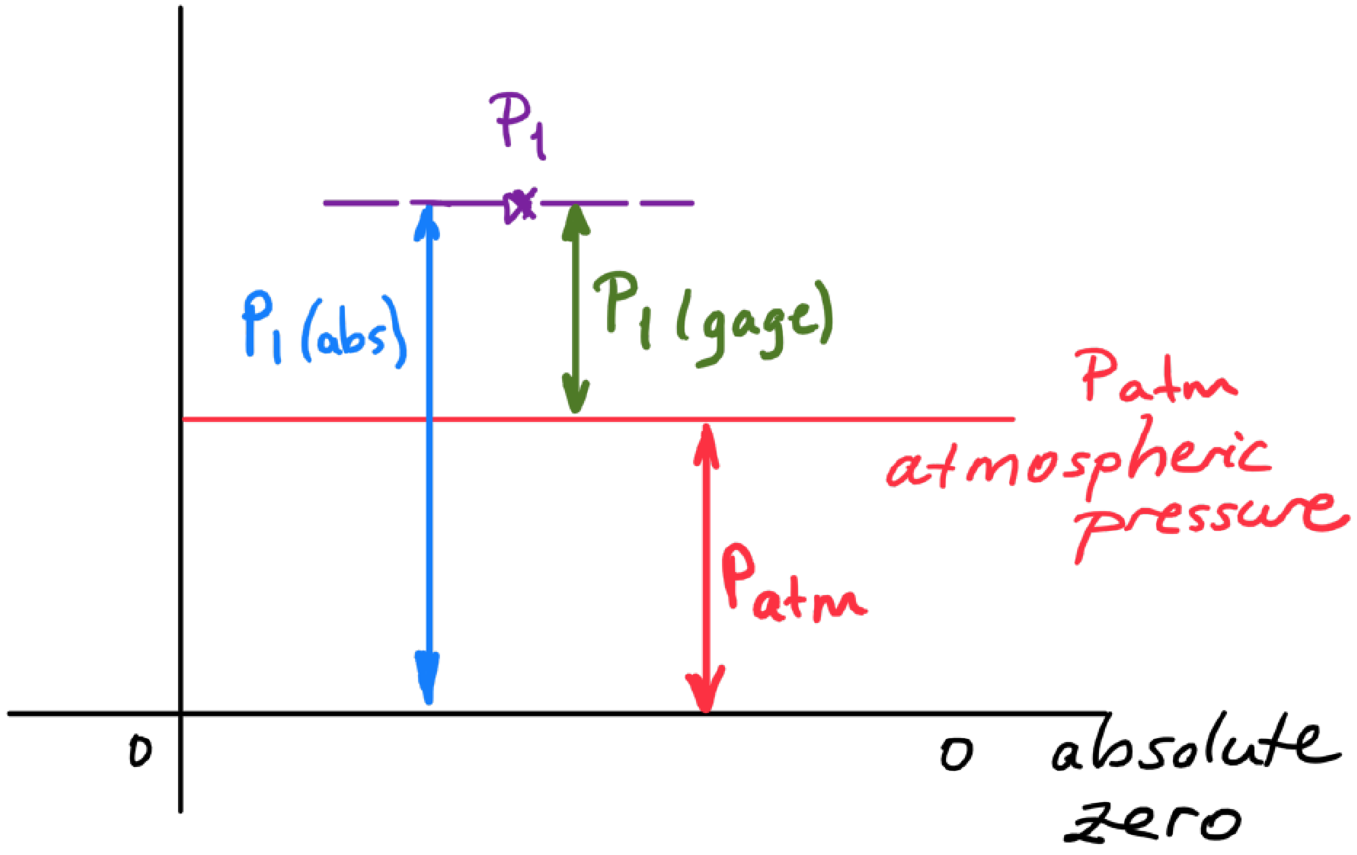


Absolute pressure is measured relative to a perfect vacuum (absolute zero pressure)

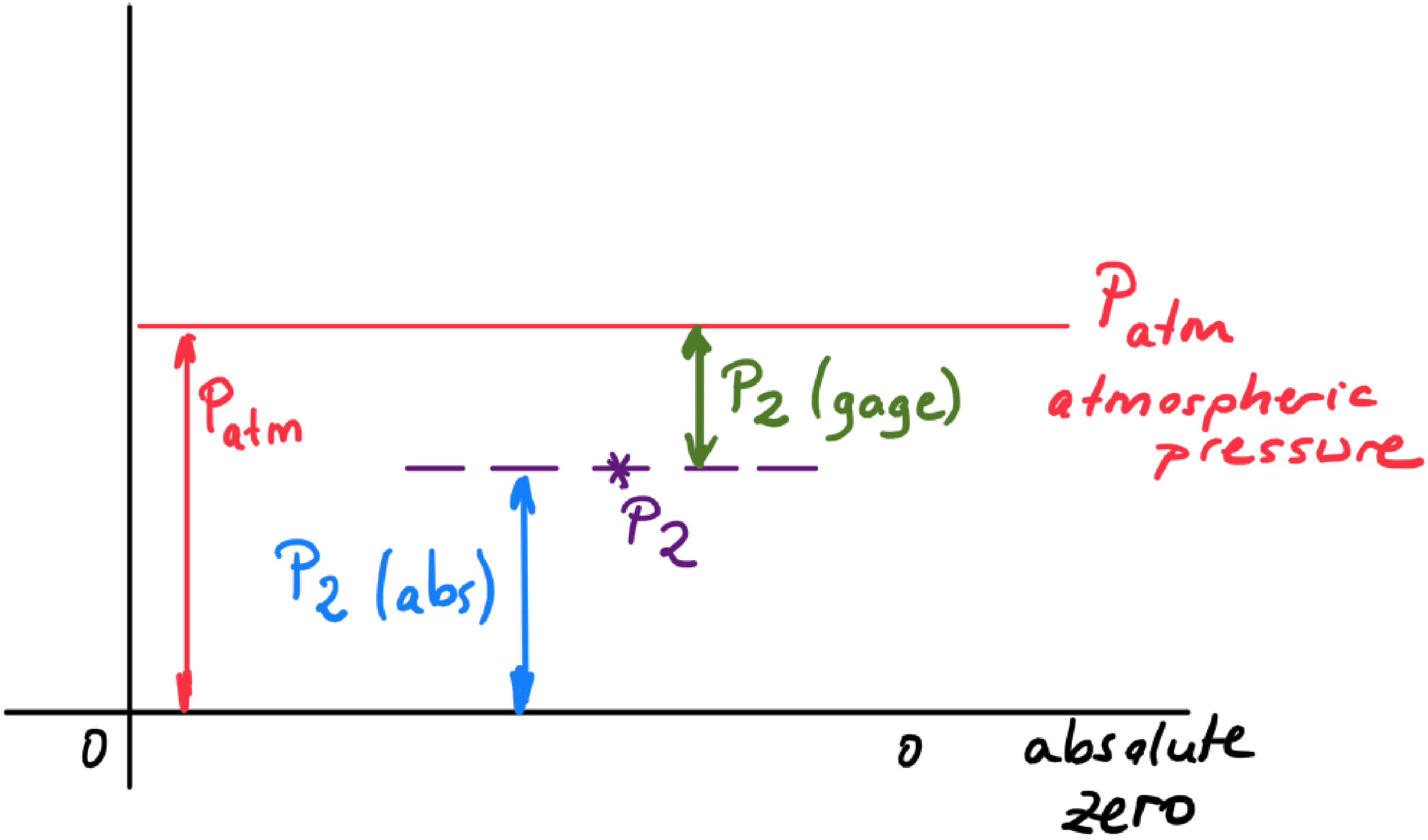
Absolute Pressure & Gage Pressure



Absolute Pressure & Gage Pressure

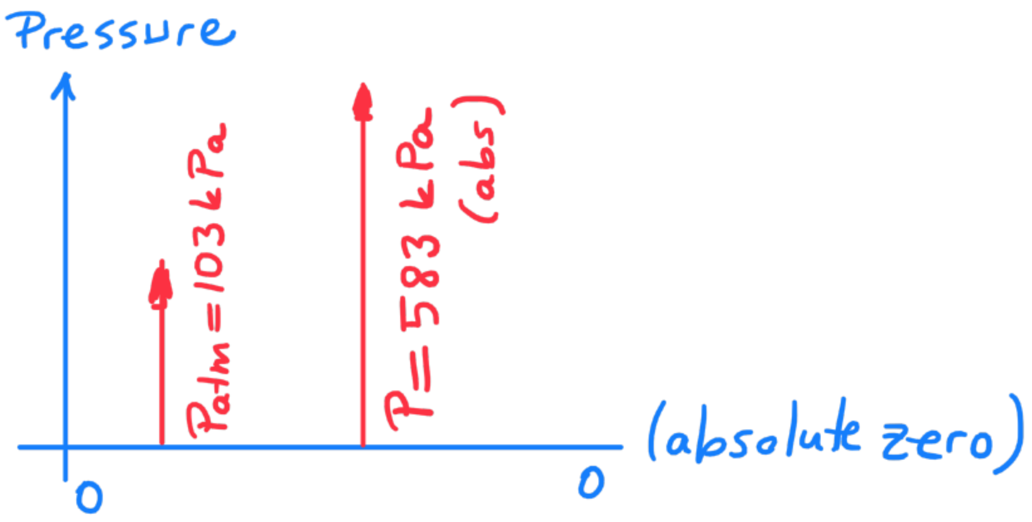


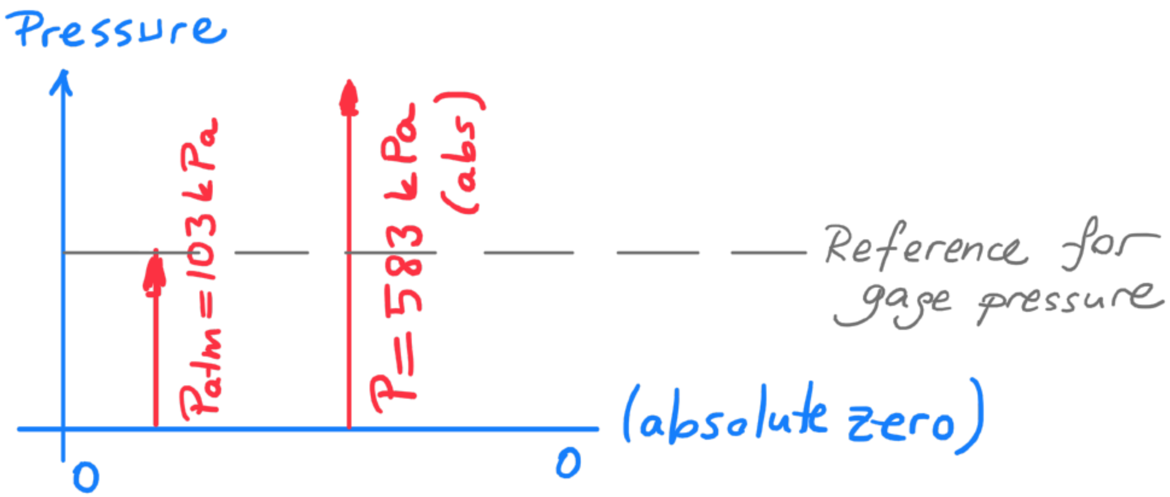
Absolute Pressure & Gage Pressure



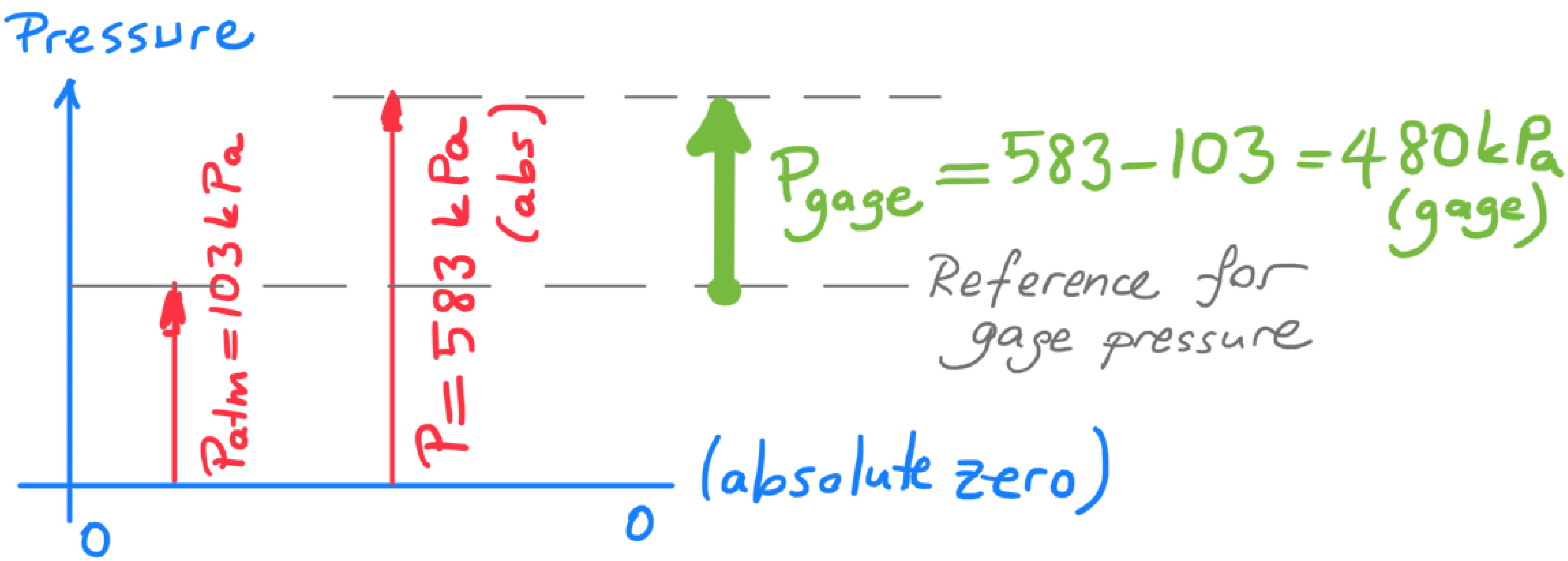
Given Pressure		p_{atm}		Express Result As:
583	kPa(abs)	103	kPa(abs)	Gage pressure

Absolute Pressure & Gage Pressure





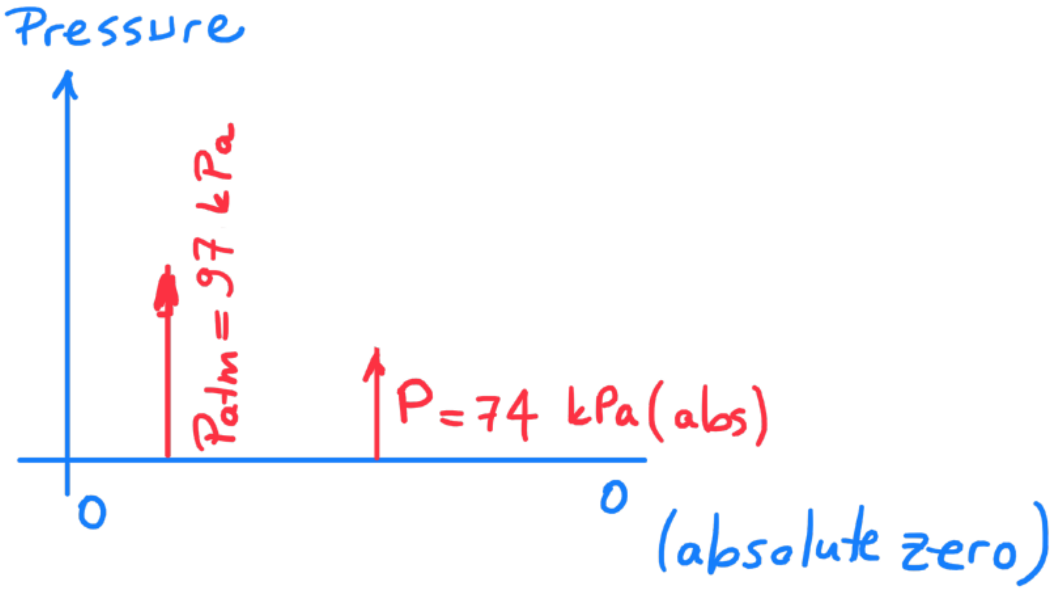
Given Pressure	p_{atm}	Express Result As:
583 kPa(abs)	103 kPa(abs)	Gage pressure

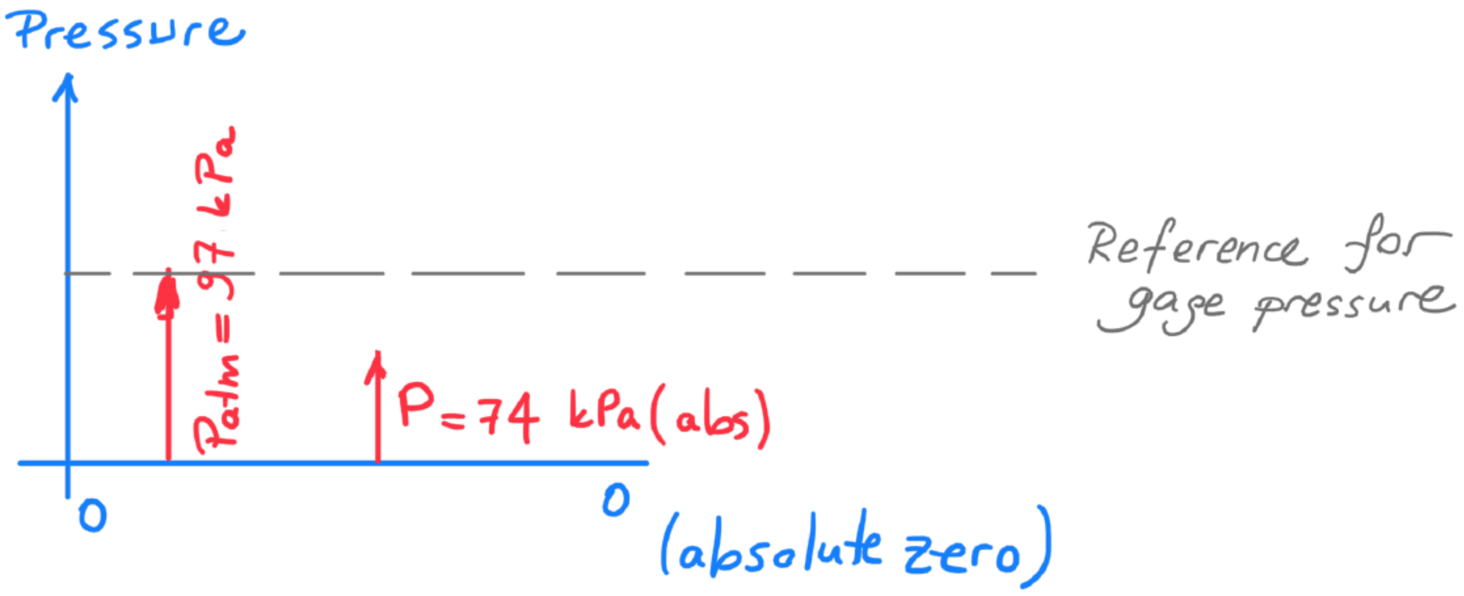


Absolute Pressure & Gage Pressure

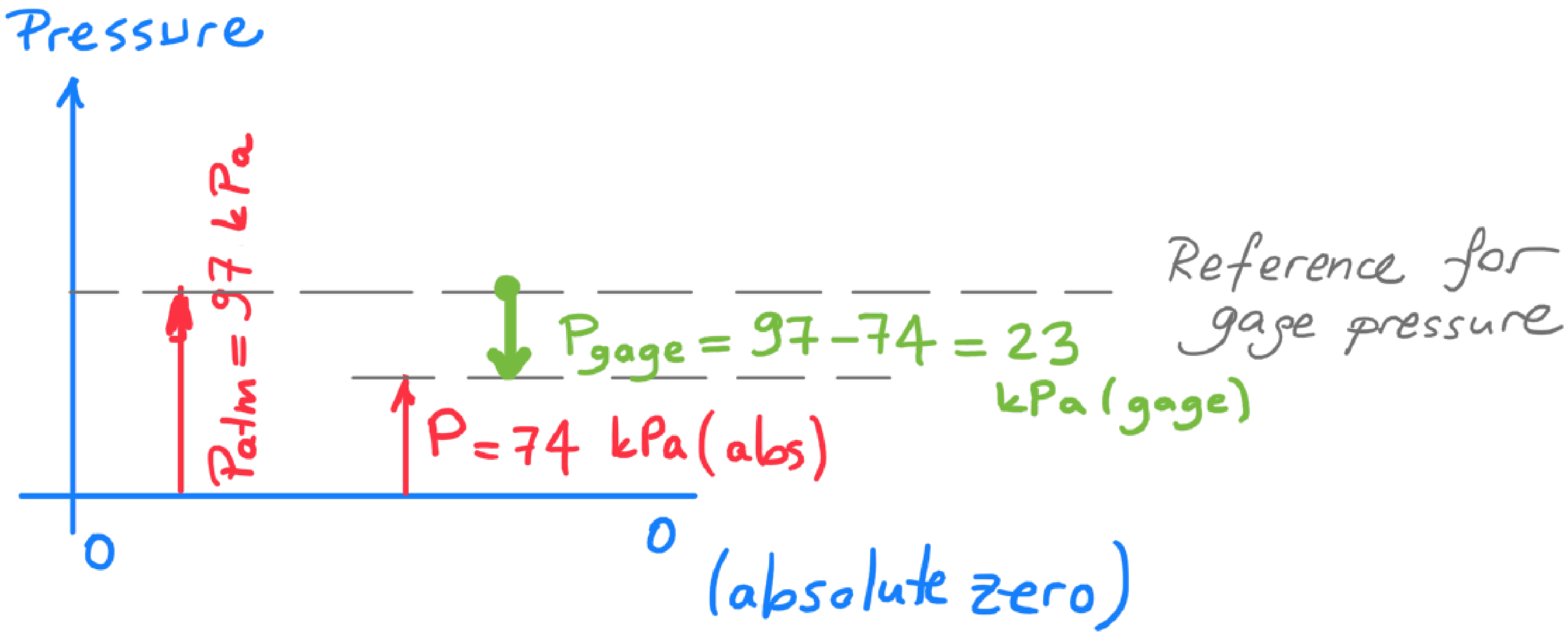
Absolute Pressure & Gage Pressure

Given Pressure	p_{atm}	Express Result As:
74 kPa(abs)	97 kPa(abs)	Gage pressure





Given Pressure		p_{atm}		Express Result As:
74	kPa(abs)	97	kPa(abs)	Gage pressure



Absolute Pressure & Gage Pressure