ENVE2061
Basic Fluid Mechanics

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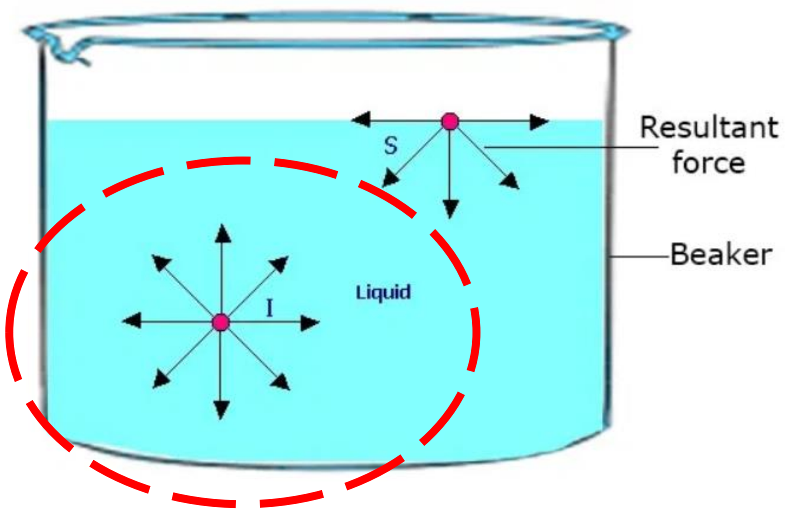
Surface Tension & Capillarity

Compressibility of Fluids

Vapor Pressure

Surface Tension and Capillarity

Even at a small distance below the surface of a liquid body, liquid molecules are attracted to each other by equal forces in all directions.



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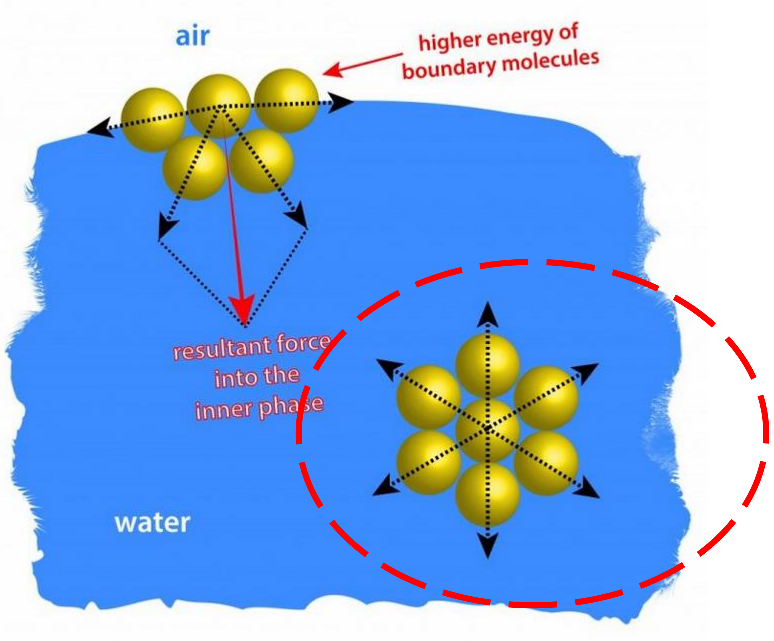
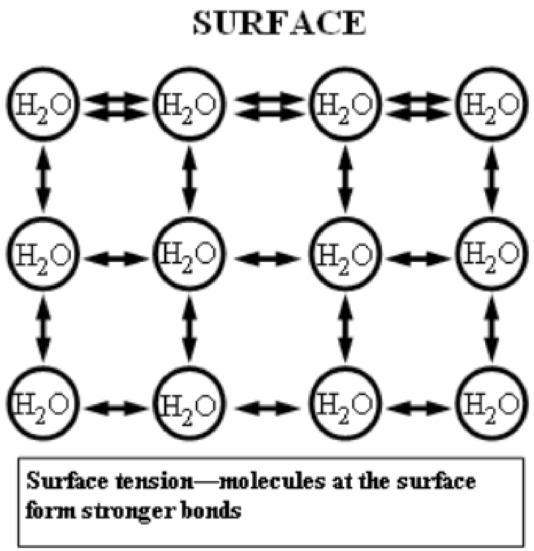


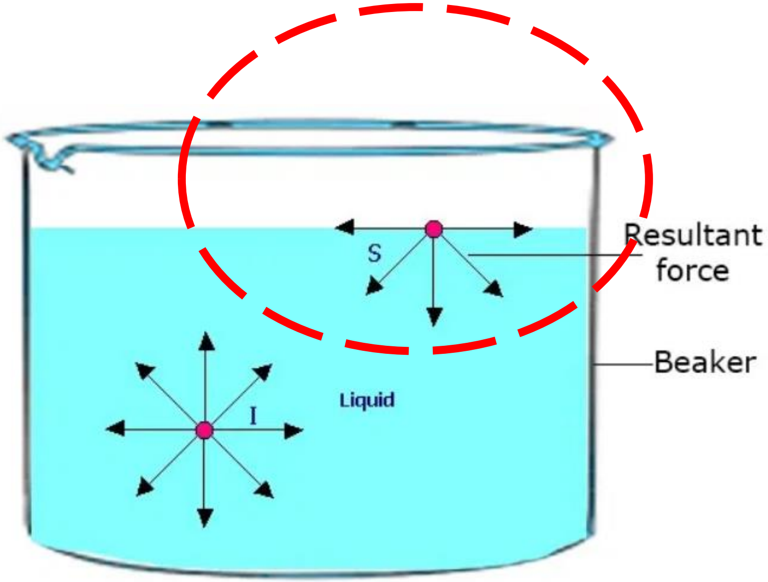
https://www.scienceabc.com/pure-sciences/what-is-surface-tension-definition-causes-examples.html

Surface Tension and Capillarity

The molecules on the surface, however, are not able to bond in all directions and therefore form stronger bonds with adjacent liquid molecules.

This causes the **liquid surface to seek a minimum possible area** by exerting surface tension tangent to the surface over the entire surface area.





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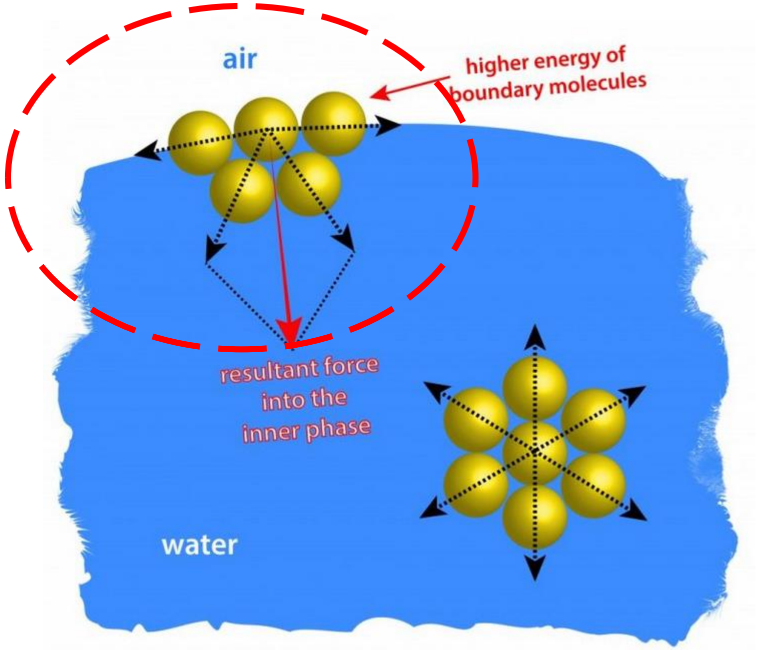

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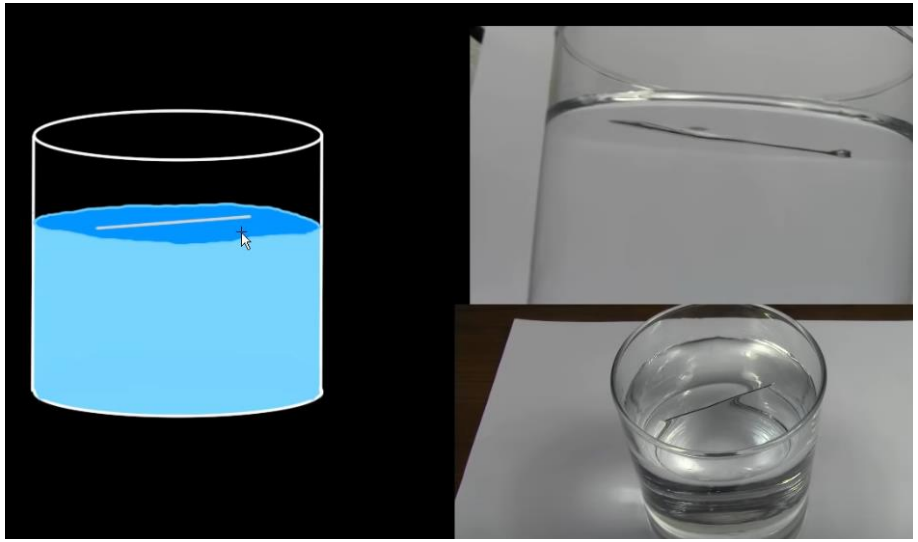
Surface Tension and Capillarity

A steel needle or a razor blade will float on water if placed gently on the surface because the tension developed
in the hypothetical skin supports it.

It seems to defy the laws of physics, but a paper clip made of steel can indeed float on the water surface. The high surface tension helps the paper clip - with much higher density - float on the water.

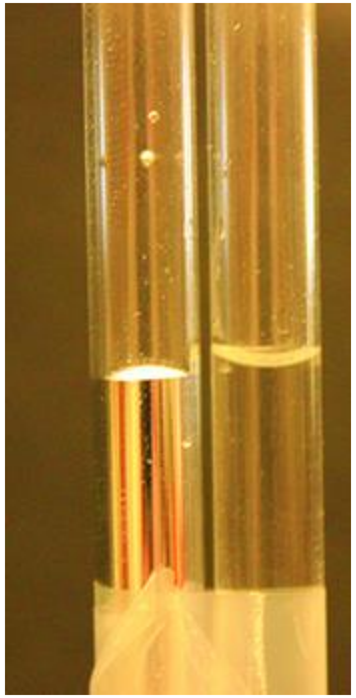
https://www.usgs.gov/special-topic/water-science-school/science/surface-tension-and-water?qt-science_center_objects=0#qt-science_center_objects



https://www.youtube.com/watch?v=pmagWO-kQ0M

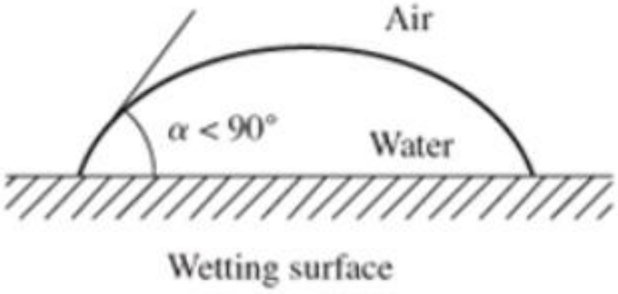
Surface Tension and Capillarity

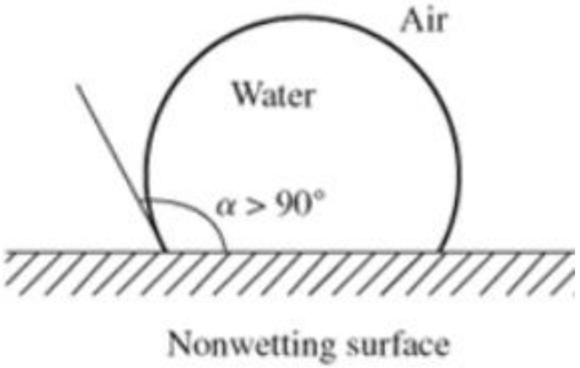
The spherical shape of dewdrops, and the rise and fall of liquid in capillary tubes are the results of surface tension.



Surface Tension and Capillarity

Most liquids adhere to solid surfaces.
The adhesive force varies, depending on the nature of the liquid and of the solid surface.

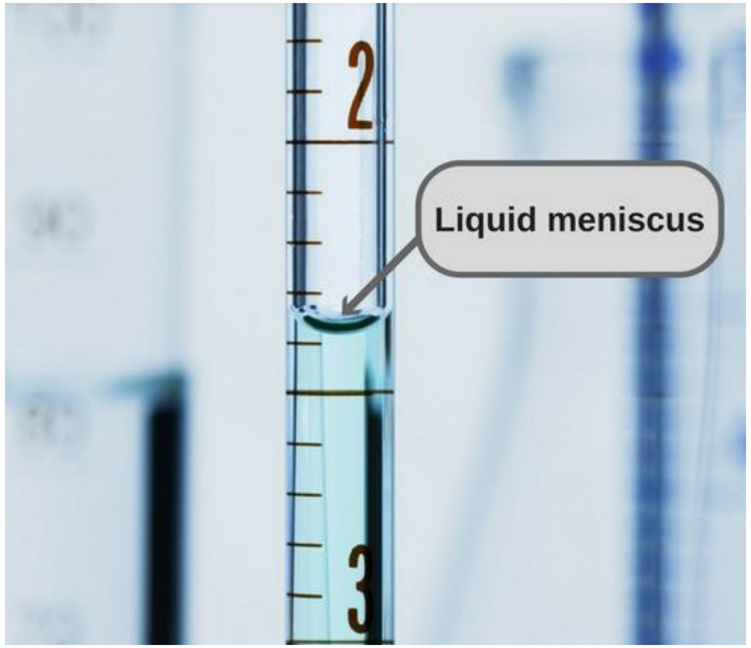




If the adhesive force between the liquid and the solid surface is greater than the cohesion in the liquid molecules, then the **liquid** tends to **spread over** and **wet the surface**

If the cohesion is greater, then a **small drop forms**

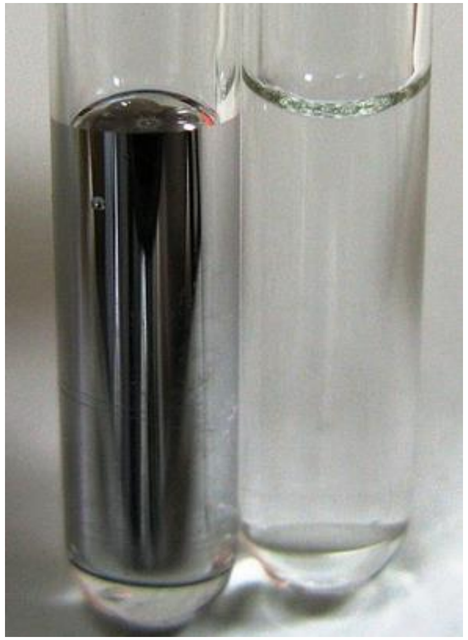
Surface Tension and Capillarity



Water wets the surface of glass, but mercury does not.

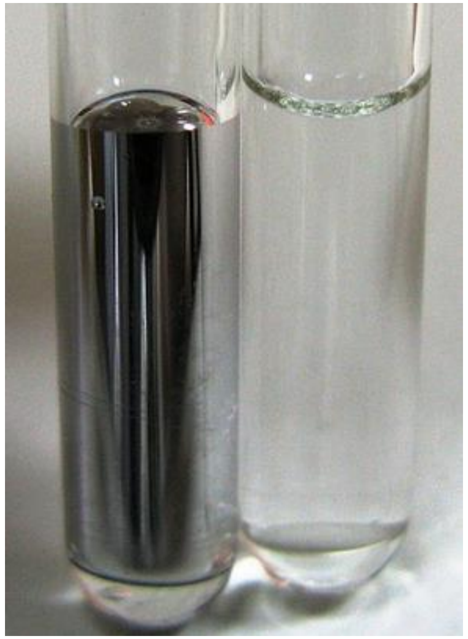
If we place a small-bore vertical glass tube into the free surface of water, the water surface in the tube rises.

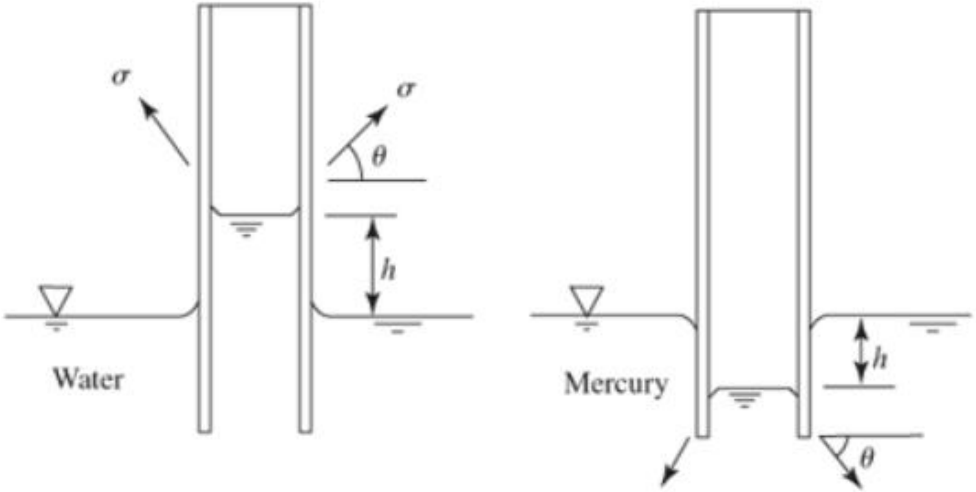
The same experiment performed with mercury will show that the mercury falls.



The phenomenon is commonly known as **capillary action**.

Surface Tension and Capillarity





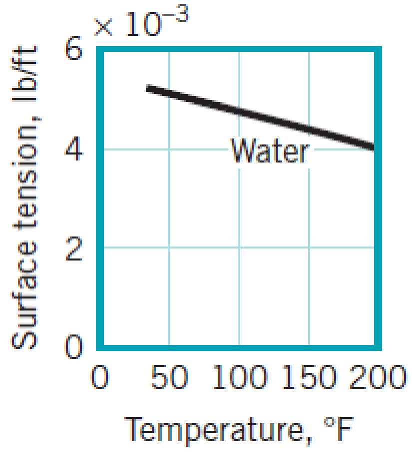
The magnitude of the capillary rise (or depression), h, is determined by the **balance of adhesive force between the liquid and solid surface** and the **weight of the liquid column above (or below) the liquid-free surface**.

Surface Tension and Capillarity

The intensity of the molecular attraction per unit length along any line in the surface is called the ***surface tension***.

$$\boldsymbol{\sigma}: \boldsymbol{surface\ tension}$$

For a given liquid the surface tension depends on temperature as well as the other fluid it is in contact with at the interface.



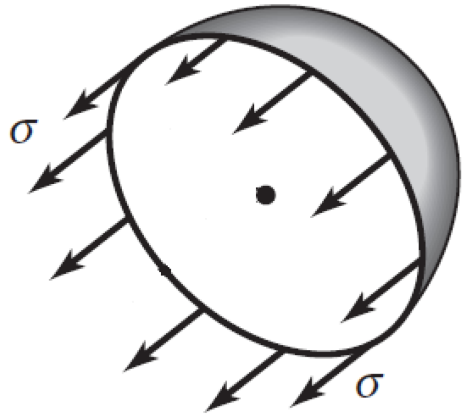
The dimensions of surface tension [F/L]
in SI: N/m
in BG: lb/ft

surface tension decreases as the temperature increases

Surface Tension of Water

	Temperature (°C/°F)									
Surface Tension, σ	0	10	20	30	40	50	60	70	80	90
	32	50	68	86	104	122	140	158	176	194
$\times 10^{-2}$ N/m	7.416	7.279	7.132	6.975	6.818	6.786	6.611	6.436	6.260	6.071
$\times 10^{-3}$ lb/ft	5.081	4.987	4.887	4.779	4.671	4.649	4.530	4.410	4.289	4.160

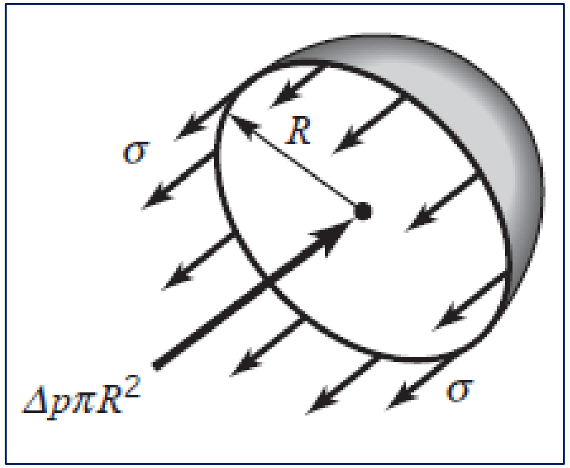
Surface Tension and Capillarity



The pressure inside a drop of fluid can be calculated using the free-body diagram shown in figure.

If the spherical drop is cut in half, the force developed around the edge due to surface tension is $2\pi R\boldsymbol{\sigma}$ (surface tension: N/m).

This force must be balanced by the pressure difference Δp, between the internal pressure (p_i), and the external pressure (p_e), acting over the circular area (πR^2)



$$2\pi R\sigma = \Delta p \pi R^2$$

$$\Delta p = p_i - p_e = \frac{2\sigma}{R}$$

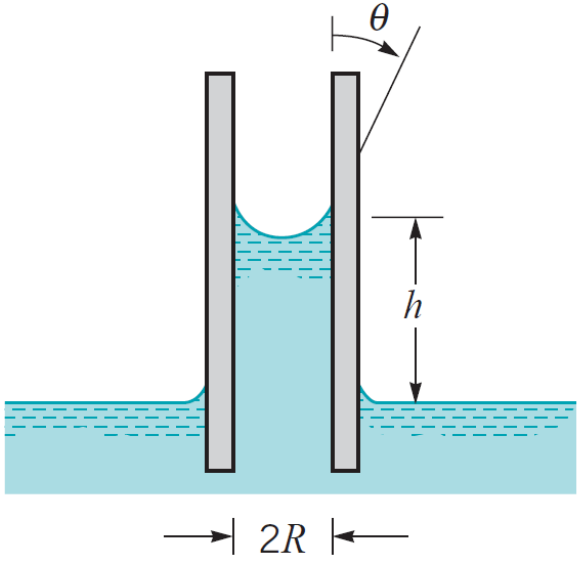
pressure inside the drop is greater than the pressure surrounding the drop

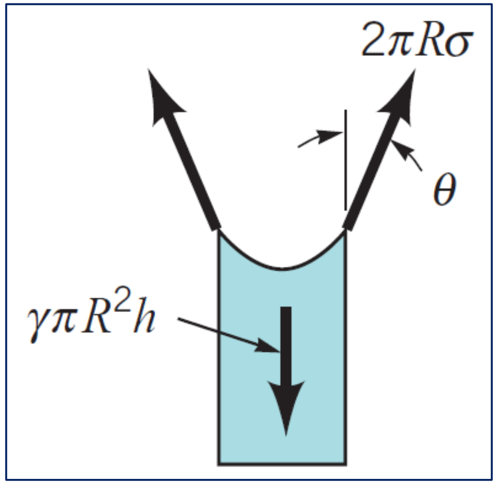
Surface Tension and Capillarity

<u>Rise or fall of a liquid in a capillary tube:</u>
Insert a small open tube into water → water level in the tube will rise above the water level outside the tube.

Attraction (adhesion) between the wall of the tube and liquid molecules which is strong enough to overcome the mutual attraction (cohesion) of the molecules and pull them up the wall.

The liquid is said to *wet* the solid surface.





Vertical force due to surface tension: $2\pi R\sigma cos\theta$
Weight: (specific weight) x (volume): $\gamma\pi R^2 h$

$$\gamma\pi R^2 h = 2\pi R\sigma cos\theta$$

$$h = \frac{2\sigma cos\theta}{\gamma R}$$

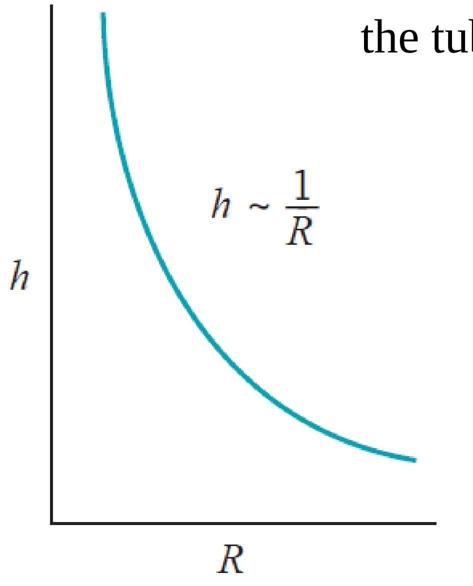
The angle of contact is a function of both the liquid and the surface. For water in contact with clean glass $\theta = 0^o$

$$h = \frac{2\sigma}{\gamma R}$$

Surface Tension and Capillarity

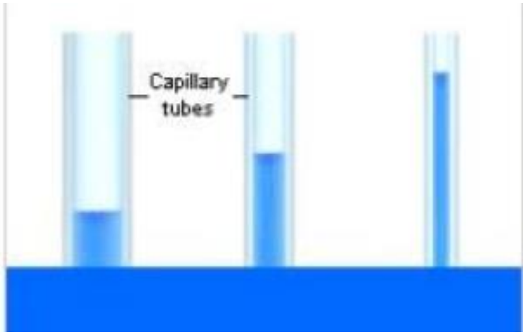
Height is inversely proportional to the tube radius.

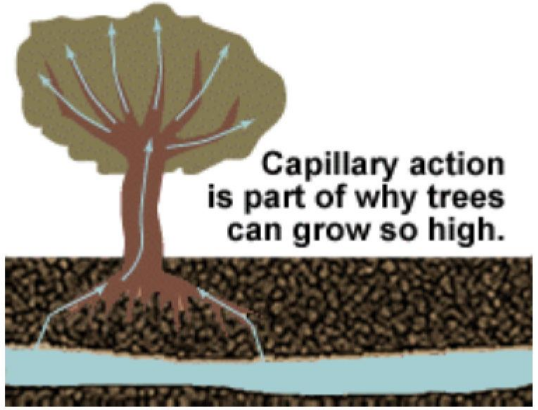
Rise of a liquid in a tube as a result of capillary action becomes increasingly pronounced as the tube radius is decreased.



$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

$$\boldsymbol{h} = \frac{2\sigma}{\gamma \boldsymbol{R}}$$

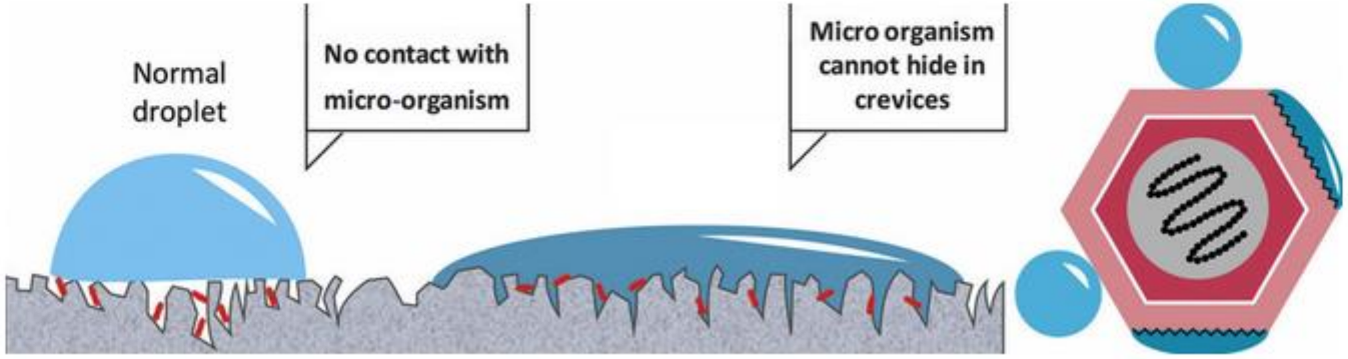




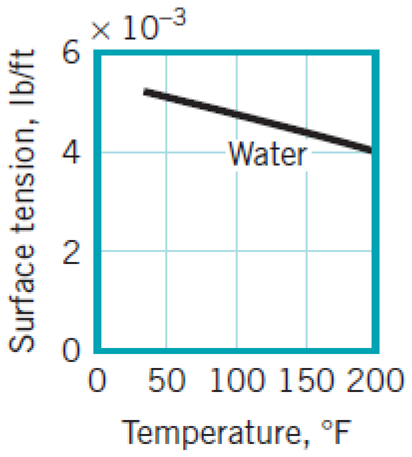
Surface Tension

Surface tension disinfectants: Disinfectants are usually solutions of low surface tension. This allow them to spread out on the cell walls of bacteria and disrupt them.

Soaps and detergents: These help the cleaning of clothes by lowering the surface tension of the water so that it more readily soaks into pores and soiled areas.



Surface Tension



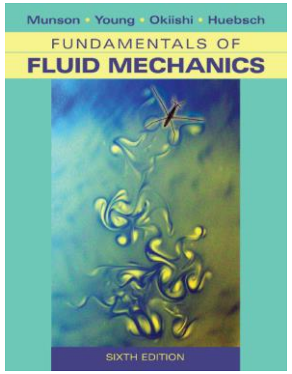
Washing with cold water: The major reason for using hot water for washing is that its surface tension is lower and it is a better wetting agent. But if the detergent lowers the surface tension, the heating may be unnecessary.

Why bubbles are round: The surface tension of water provides the necessary wall tension for the formation of bubbles with water. The tendency to minimize that wall tension pulls the bubbles into spherical shapes.

EXAMPLE 1.8 Capillary Rise in a Tube

GIVEN Pressures are sometimes determined by measuring the height of a column of liquid in a vertical tube.

FIND What diameter of clean glass tubing is required so that the rise of water at 20 °C in a tube due to capillary action (as opposed to pressure in the tube) is less than $h = 1.0$ mm?



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FIND What diameter of clean glass tubing is required so that the rise of water at 20 °C in a tube due to capillary action (as opposed to pressure in the tube) is less than $h = 1.0$ mm?

SOLUTION

From Eq. 1.22

$$h = \frac{2\sigma \cos\theta}{\gamma R}$$

so that

$$R = \frac{2\sigma \cos\theta}{\gamma h}$$

For water at 20 °C (from Table B.2), $\sigma = 0.0728$ N/m and $\gamma = 9.789$ kN/m^3. Since $\theta \approx 0°$ it follows that for $h = 1.0$ mm,

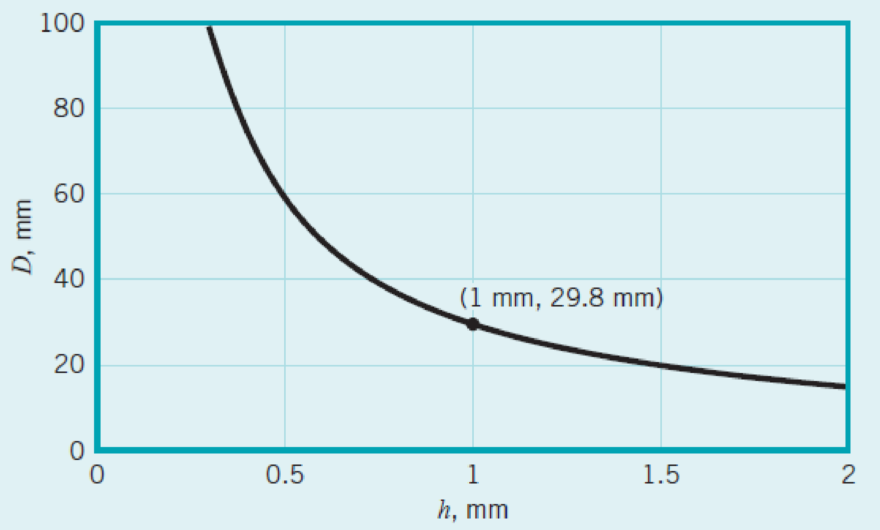
$$R = \frac{2(0.0728 \text{ N/m})(1)}{(9.789 \times 10^3 \text{ N/m}^3)(1.0 \text{ mm})(10^{-3} \text{ m/mm})} = 0.0149 \text{ m}$$

and the minimum required tube diameter, D, is

$$D = 2R = 0.0298 \text{ m} = 29.8 \text{ mm}$$ **(Ans)**

COMMENT By repeating the calculations for various values of the capillary rise, h, the results shown in Fig. E1.8 are obtained.

Note that as the allowable capillary rise is decreased, the diameter of the tube must be significantly increased. There is always some capillarity effect, but it can be minimized by using a large enough diameter tube.

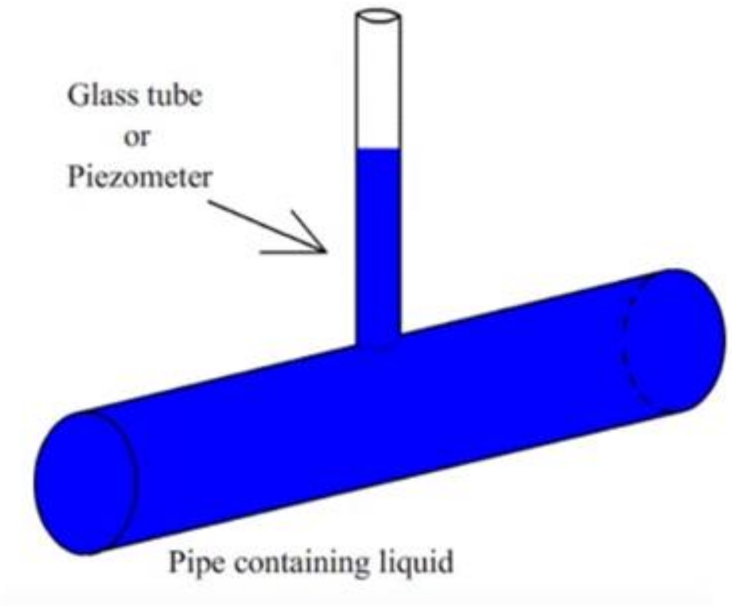


1.5.5. Vertical glass tubes (piezometers) can be used to measure the pressure in pipes. However, capillary action can create a measurement error. Determine the error (in cm) resulting from the use of 1.2-cm-diameter piezometers in a pipe conveying saltwater $(\mathrm{SG} = 1.03)$ that has a surface tension 20% greater than fresh water $(T = 35^{\circ}\mathrm{C})$ if the contact angle is 35°.

For fresh water:

σ = 6.90 x 10 $^{-2}$ N/m

γ = 9750 N/m^3



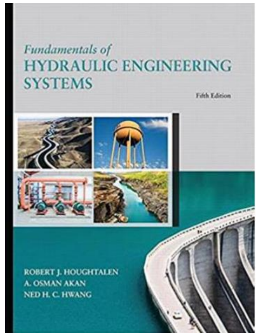
$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

Capillary rise

σ = 6.90 x 10 $^{-2}$ * 1.2 = 8.28 x 10 $^{-2}$ N/m

γ = 9750 * 1.03 = 1.00 x 10^4 N/m^3

$$h = 1.58x10^{-3}\ m = 0.158\ cm$$

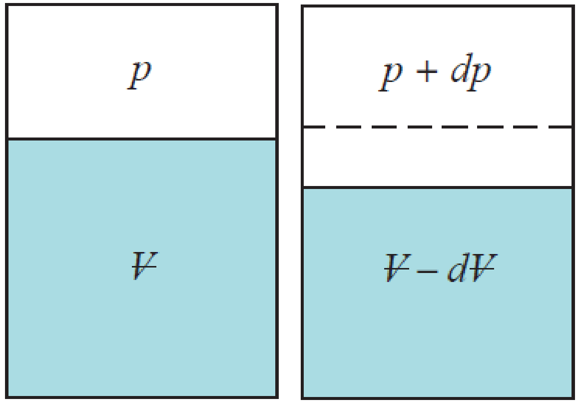


Compressibility of Fluids

How easily can the volume (and thus the density) of a given mass of the fluid be changed when there is a change in pressure?

: How compressible is the fluid?

A property that is commonly used to characterize compressibility is the ***bulk modulus, E_b,*** ***(also known as bulk modulus of elasticity or volume modulus of elasticity)***



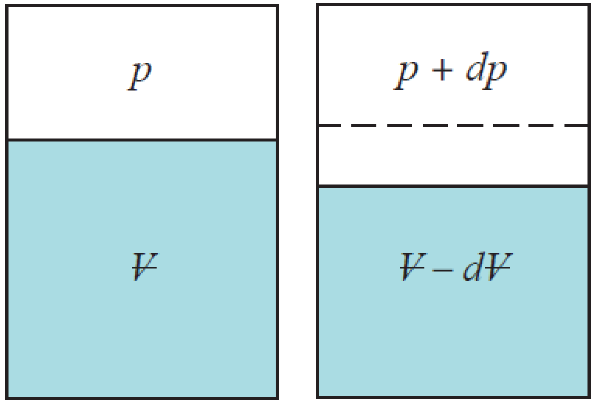
ΔP change in pressure (pressure increase)
ΔVol change in volume (pressure increase cause the volume to decrease)

$$\Delta P = -E_b\left(\frac{\Delta Vol}{Vol}\right)$$

Compressibility of Fluids

Water is commonly assumed to be **incompressible under ordinary conditions**. In reality, it is about 100 times more compressible than steel.

It is necessary to consider the compressibility of water when water hammer issues are possible.



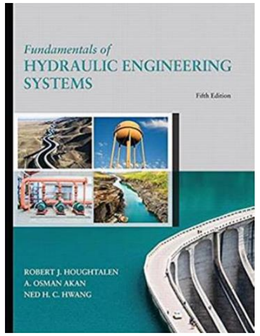
The bulk modulus of elasticity of water varies with both temperature and pressure.

$$\Delta P = -E_b\left(\frac{\Delta Vol}{Vol}\right)$$

In the range of practical applications in typical hydraulic systems, a value of $2.2\ x\ 10^9\ N/m^2$ or in BG units, $3.2\ x\ 10^5\ lb/in^2(psi)$ can be used.

Example 1.3

The density of seawater is $1{,}026\ \mathrm{kg/m^3}$ at sea level. Determine the density of seawater on the ocean floor 2,000 m deep, where the pressure is approximately $2.02 \times 10^7\ \mathrm{N/m^2}$.



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Solution

The change of pressure at a depth of 2,000 m from the pressure at the water surface is

$$\Delta P = P - P_{\text{atm}} = 2.01 \times 10^7\ \mathrm{N/m^2}$$

Standard sea-level atmospheric pressure (by international agreement) is **14.696 psi (abs)** or **101.33 kPa (abs)**. For most calculations these pressures can be rounded to 14.7 psi and 101 kPa, respectively.

$$\Delta P = -E_b\left(\frac{\Delta Vol}{Vol_0}\right)$$

$$\left(\frac{\Delta Vol}{Vol_0}\right) = \left(\frac{-\Delta P}{E_b}\right) = \frac{-2.01 \times 10^7}{2.20 \times 10^9} = -0.00914$$

$$\rho = \left(\frac{m}{Vol}\right) \quad \therefore Vol = \left(\frac{m}{\rho}\right)$$

$$\Delta Vol = \left(\frac{m}{\rho}\right) - \left(\frac{m}{\rho_0}\right) \quad \therefore \frac{\Delta Vol}{Vol_0} = \left(\frac{\rho_0}{\rho}\right) - 1$$

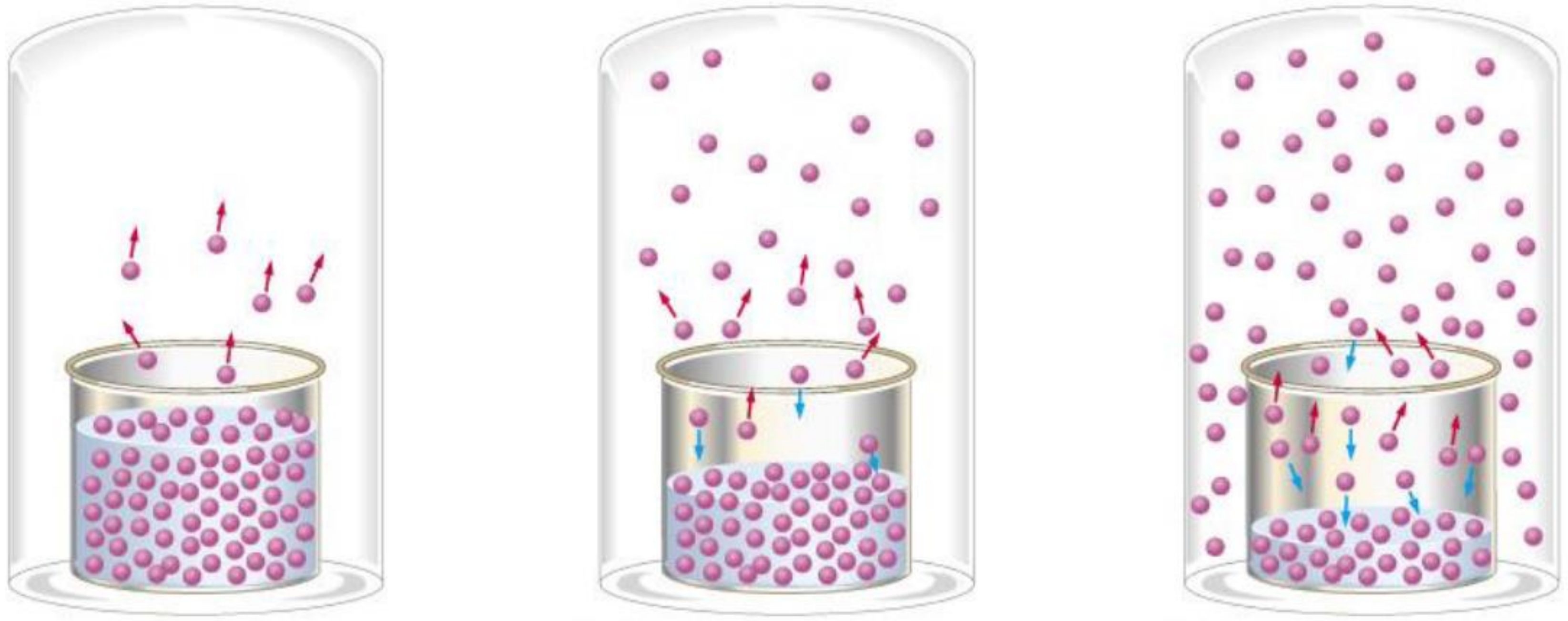
$$\rho = \left(\frac{\rho_0}{1 + \frac{\Delta Vol}{Vol_0}}\right) = \left(\frac{1{,}026\ \mathrm{kg/m^3}}{1 - 0.00914}\right) = 1{,}040\ \mathrm{kg/m^3}$$

Vapor Pressure

It is a common observation that liquids such as water and gasoline will evaporate if they are simply placed in a container open to the atmosphere

Evaporation takes place because some liquid molecules at the surface have sufficient momentum to overcome the intermolecular cohesive forces and escape into the atmosphere.

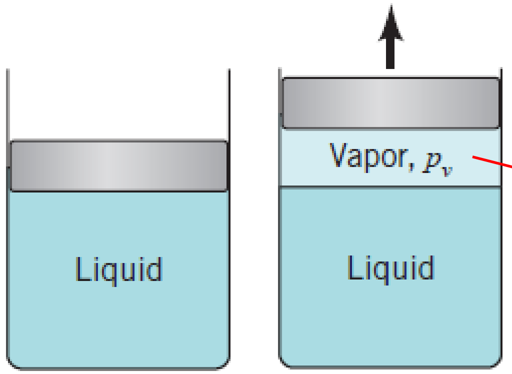
Vapor Pressure



If the container is closed with a small air space left above the surface, and this space evacuated to form a vacuum, a pressure will develop in the space as a result of the vapor that is formed by the escaping molecules.

When an equilibrium condition is reached so that the number of molecules leaving the surface is equal to the number entering, the vapor is said to be saturated and the pressure that the vapor exerts on the liquid surface is termed the ***vapor pressure***

Vapor Pressure



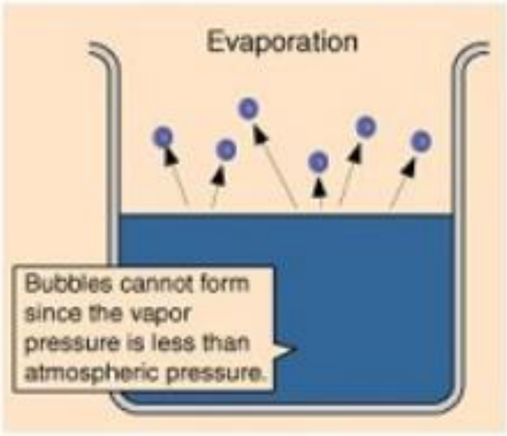
the space between the liquid and the end becomes filled with vapor at a pressure equal to the vapor pressure.

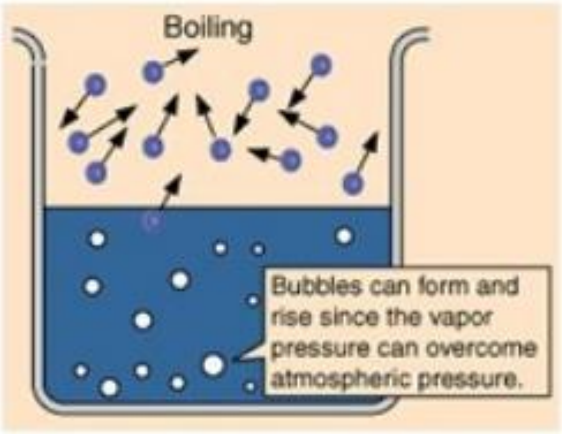
Similarly, if the end of a completely liquid-filled container is moved as shown in the figure without letting any air into the container

Since the development of a **vapor pressure** is closely associated with molecular activity, the value of vapor pressure for a particular liquid **depends on temperature**

Vapor Pressure

Boiling, which is the formation of vapor bubbles within a fluid mass, is **initiated when the absolute pressure in the fluid reaches the vapor pressure**.





At standard atmospheric pressure, water will boil when the temperature reaches 100 °C

Vapor pressure of water at 100 °C is 101 kPa (14.7 psi)

If we attempt to boil water at a higher elevation (such as 30,000 ft above the sea level: ~ elevation of Mt Everest)

Atmospheric pressure is 4.37 psi at this elevation.
Boiling start when the temperature is about 70 °C

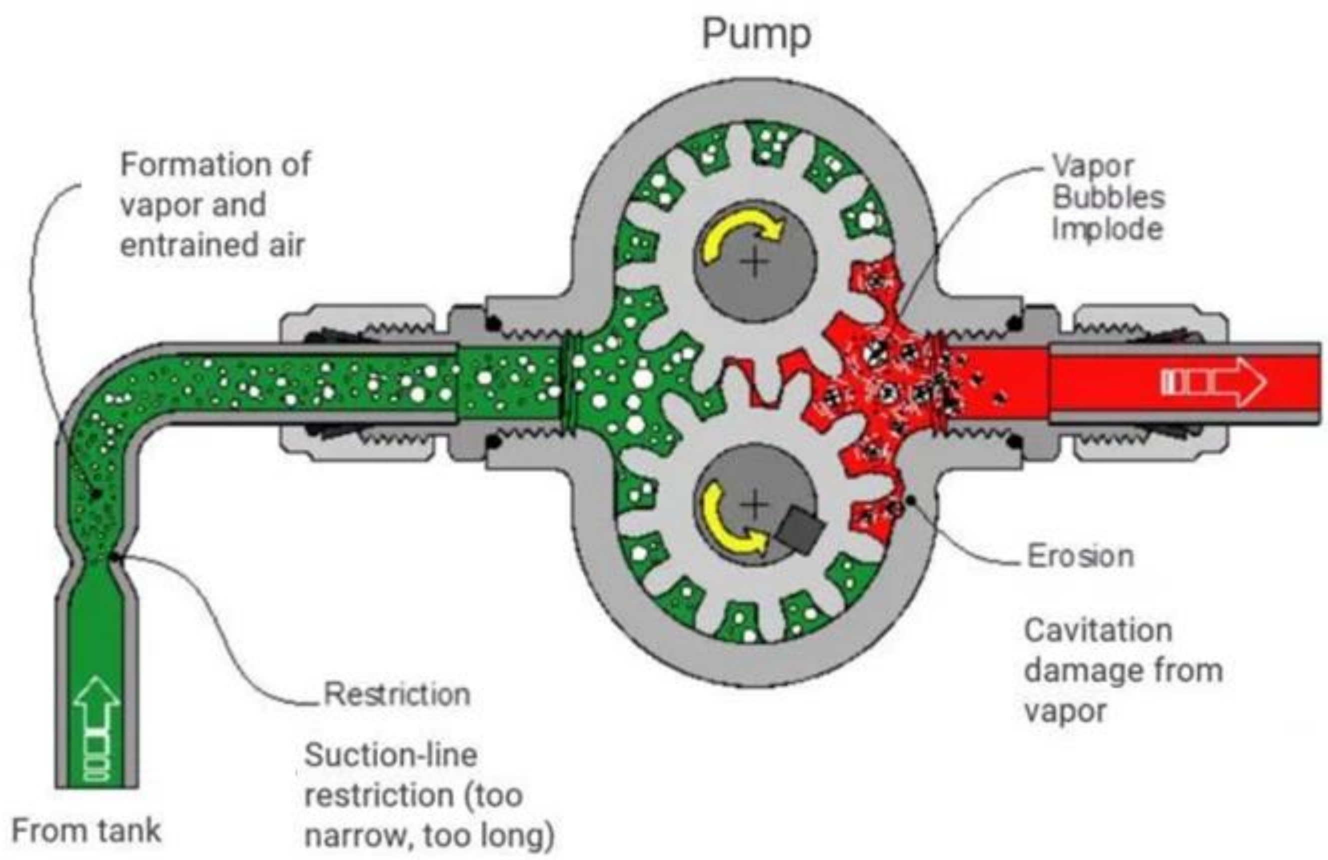
Boiling can be started at a given pressure acting on the fluid by raising the temperature, or at a given fluid temperature by lowering the pressure.

Vapor Pressure - Cavitation

This phenomenon may occur in flow through the irregular, narrowed passages of a valve or pump.

When vapor bubbles are formed in a flowing fluid, they are swept along into regions of higher pressure where they suddenly collapse with sufficient intensity to actually cause structural damage.

The formation and subsequent collapse of vapor bubbles in a flowing fluid, called ***cavitation***



Section 1.8 Vapor Pressure

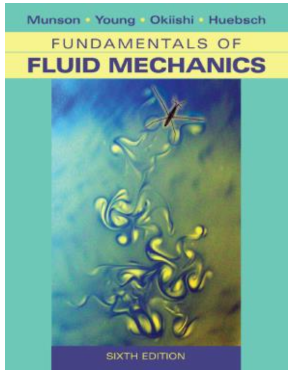
1.88 During a mountain climbing trip it is observed that the water used to cook a meal boils at 90 °C rather than the standard 100 °C at sea level. At what altitude are the climbers preparing their meal? (See Tables B.2 and C.2 for data needed to solve this problem.)

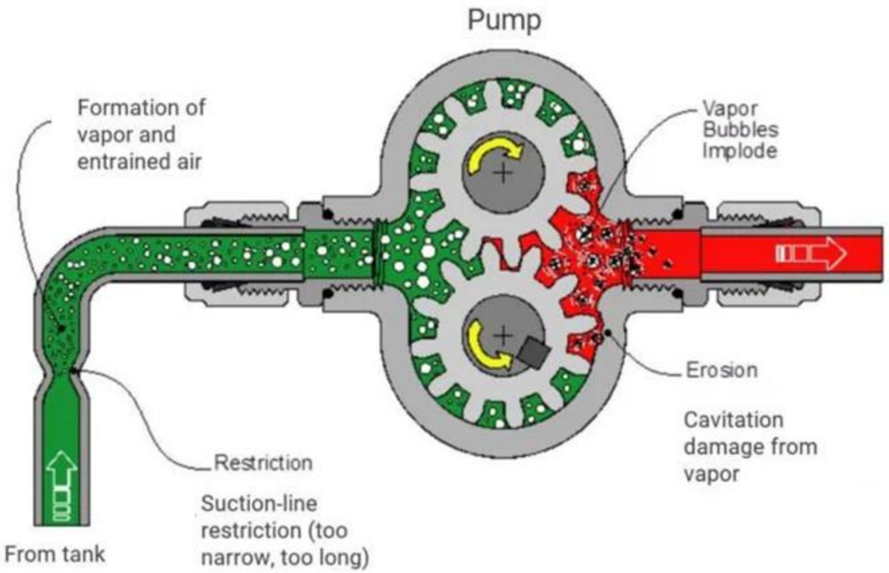
1.89 When a fluid flows through a sharp bend, low pressures may develop in localized regions of the bend. Estimate the minimum absolute pressure (in psi) that can develop without causing cavitation if the fluid is water at 160 °F.

1.90 Estimate the minimum absolute pressure (in pascals) that can be developed at the inlet of a pump to avoid cavitation if the fluid is carbon tetrachloride at 20 °C.

1.91 When water at 70 °C flows through a converging section of pipe, the pressure decreases in the direction of flow. Estimate the minimum absolute pressure that can develop without causing cavitation. Express your answer in both BG and SI units.

1.92 At what atmospheric pressure will water boil at 35 °C? Express your answer in both SI and BG units.



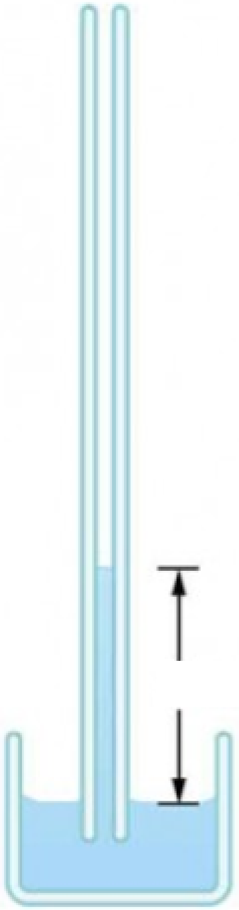


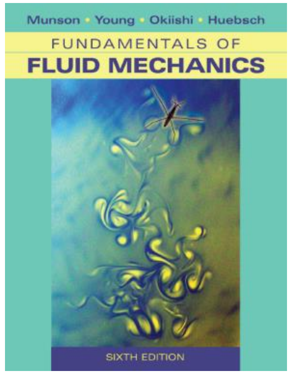
Section 1.9 Surface Tension

1.94 When a 2-mm-diameter tube is inserted into a liquid in an open tank, the liquid is observed to rise 10 mm above the free surface of the liquid. The contact angle between the liquid and the tube is zero, and the specific weight of the liquid is 1.2×10^4 N/m^3. Determine the value of the surface tension for this liquid.

1.95 Small droplets of carbon tetrachloride at 68 °F are formed with a spray nozzle. If the average diameter of the droplets is 200 μm, what is the difference in pressure between the inside and outside of the droplets?

1.98 To measure the water depth in a large open tank with opaque walls, an open vertical glass tube is attached to the side of the tank. The height of the water column in the tube is then used as a measure of the depth of water in the tank. (**a**) For a true water depth in the tank of 3 ft, make use of Eq. 1.22 (with $\theta \simeq 0°$) to determine the percent error due to capillarity as the diameter of the glass tube is changed. Assume a water temperature of 80 °F. Show your results on a graph of percent error versus tube diameter, D, in the range 0.1 in. $< D <$ 1.0 in. (**b**) If you want the error to be less than 1%, what is the smallest tube diameter allowed?





Section 1.7 Compressibility of Fluids

1.76 Estimate the increase in pressure (in psi) required to decrease a unit volume of mercury by 0.1%.

1.77 A 1-m^3 volume of water is contained in a rigid container. Estimate the change in the volume of the water when a piston applies a pressure of 35 MPa.